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MERGING OF GALAXIES WITH CENTRAL BLACK HOLES. II. EVOLUTION OF THE BLACK HOLE BINARY AND THE STRUCTURE OF THE CORE

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ABSTRACT

We investigated the evolution of the black hole binary formed by the merging of two galaxies, each containing a central massive black hole. Our main goal here is to determine if the black hole binary can merge through the hardening by dynamical friction and the gravitational wave radiation. We performed N-body simulations of the merging of two galaxies with a wide range of the total number of particles to investigate the effect of the number of particles on the evolution of the black hole binary. We found that the evolution timescale was independent of the number of particles in the galaxy N until the semimajor axis reached a critical value. After the semimajor axis became smaller than this critical value, the evolution timescale was longer for larger numbers of particles. Qualitatively, this behavior is understood naturally as the result of the "loss cone" effect. However, the dependence of the timescale on N is noticeably weaker than the theoretical prediction. In addition, the critical semimajor axis is smaller than the theoretical prediction. The timescale of evolution through gravitational radiation at this critical semimajor axis is longer than the Hubble time. We discuss the reason of this discrepancy and the implication of the present result on the structure of the elliptical galaxies and QSO activities.

Subject headings: black hole physics — galaxies: active — galaxies: interactions — galaxies: nuclei — methods: numerical

1. INTRODUCTION

The possibility of the formation of massive black hole (BH) binaries in the cores of elliptical galaxies was first pointed out by Begelman, Blandford, & Reese (1980, hereafter BBR). Their argument is summarized as follows. The energy source for active galactic nucleus (AGN) or QSO activities is most likely to be a massive BH with a mass $M_{\rm BH} \simeq 10^8\ M_\odot$. If two galaxies with central massive black holes merge with each other, BHs sink toward the center of the merger remnant because of the dynamical friction from stars, and they form a binary.

BBR argued that such a binary would have a typical lifetime larger than the Hubble time, since the BH binary would create the "loss cone" in the distribution of field stars. The stars can enter this loss cone region only through the change of their orbits through encounters with other stars. Therefore, the timescale of the repopulation of the loss cone is the two-body relaxation timescale of the stars in the core.

According to their argument, the evolution the binary proceeds in the two-body relaxation timescale of the core, once the semimajor axis of the binary comes down to the loss cone radius, $r_{\rm lc}$, which is given by

$$r_{\rm lc} = \left(\frac{M_{\rm BH}}{M_{\rm c}}\right)^{3/4} r_{\rm c}\ , \qquad (1)$$

Here $M_{\rm BH}$ is the mass of a BH, $M_{\rm c}$ is the mass of the core, and $r_{\rm c}$ is the core radius. When the separation between the BHs becomes smaller than this $r_{\rm lc}$, the binary effectively sweeps out the stars that can interact with the binary. Further evolution of the BH binary is only through the interaction with stars that diffuse into the loss cone by two-body relaxation between stars. They concluded that the BH binary in the merger remnant has a lifetime much longer than the Hubble time.

Ebisuzaki, Makino, & Okumura (1991) pointed out that the lifetime is much shorter if the BH binary is highly eccentric. BBR assumed that the binary remains circular. The timescale of the gravitational radiation is proportional to $(1-e)^{-3.5}$ for $e \sim 1$, where e is the eccentricity. If the BH binary becomes highly eccentric, the lifetime would become very short. In fact, even for the average "thermal" eccentricity of 0.7, the lifetime is shorter than that of a circular binary by nearly 2 orders of magnitude. The evolution of the binary is driven by the dynamical friction from field stars. If we naively apply Chandrasekhar's dynamical friction formula, the eccentricity should grow quickly, since the dynamical friction is inversely proportional to the third power of the velocity and therefore the strongest at the apoastron (Fukushige, Ebisuzaki, & Makino 1992).

Mikkola & Valtonen (1992) numerically integrated the evolution of a BH binary in the core of a galaxy. They followed the orbit of BHs in the distribution of the field stars. The gravitational interactions between BHs and between field stars and BHs were directly calculated. The interactions between the field stars were approximated by a fixed potential. As a result, they could not follow the evolution of the structure of the core. However, they could use a fairly large number of particles ($N = 10{,}000$). They found that the binary hardened linearly in time. In other words, in their calculation the loss cone depletion did not take place. As for the eccentricity, they found that it increases only very slowly. The rapid increase of the eccentricity predicted by Fukushige et al. (1992) did not take place. This is not very surprising, because the standard dynamical friction formula used by Fukushige et al. (1992) is valid only for the motion of a single massive particle and therefore is not guaranteed to give a correct result for the evolution of a binary. They also determined the rate of evolution of energy and eccentricity of the BH binary by a scattering experiment. The result of their scattering experiment was consis-

tent with the result of the simulation.

Quinlan (1996) performed extensive scattering experiments of a BH binary and a field particle. He extended the work of Mikkola & Valtonen (1992) to include unequal-mass BHs. His result is similar to that of Mikkola & Valtonen (1992). The change in the eccentricity is small unless it is initially close to unity.

Makino et al. (1993) performed self-consistent N-body simulation of the evolution of a BH binary in the core of a galaxy with 16,384 particles. Their results are summarized as follows: (1) The BH binary becomes harder beyond the loss cone radius to at least $0.1r_{\rm lc}$, without showing any sign of slowing down; (2) after the binary is formed, the eccentricity of the binary remains roughly constant; and (3) the eccentricity depends strongly on the initial condition of the BHs. Thus, their result is again consistent with the result of Mikkola & Valtonen (1992) but apparently in contradiction with the theoretical prediction of BBR. Makino & Ebisuzaki (1996, hereafter referred to as Paper I) performed N-body simulations of the merging of two galaxies with central massive BHs with 32k particles, but their main interest is in the structure of the core and not the evolution of the BH binary.

In the present paper, we give the result of self-consistent direct N-body simulations with the number of particles much larger than that was employed in Paper I, Makino et al. (1993), or Mikkola & Valtonen (1992).

In order to study the evolution of the binding energy of the central BH binary, we performed the merging simulations from the same initial condition, but with several different number of particles. The small-N effect reduces the evolution timescale of the BH binary in at least two different ways. The timescale of the repopulation of the loss cone, $t_{\rm rl}$, is proportional to the core relaxation time, $t_{\rm rc}$. Therefore, if the timescale of the evolution of the BH binary, t_b, is actually determined by $t_{\rm rl}$, t_b should be proportional to $t_{\rm rc}$. In N-body simulations, however, $t_{\rm rc}$ is not much larger than the timescale of the depletion of the loss cone. If the repopulation is faster than the depletion, the loss cone is not actually formed, and the growth rate of the BH binary would be independent of N.

The center of mass of the BH binary has small random velocity, since its kinetic energy is in equipartition with those of field stars (Bahcall & Wolf 1976; Mikkola & Valtonen 1992). Thus, the BH binary might wander in, or even out of, the loss cone. As a result, the effective radius of the loss cone might become larger in N-body experiments. The random velocity would be proportional to the square root of the mass ratio between the field particles and a BH binary. Therefore, a BH binary in the numerical simulation has the random velocity several hundred times larger than that of a real BH binary.

Note that this second problem exists even in simulations with fixed field potential, such as that of Mikkola & Valtonen (1992), since in their calculation BHs felt the forces from the field particles. The force from the field particles shows the same fluctuating behavior as in self-consistent N-body calculations. Moreover, the effect of the relaxation is not completely suppressed either, because the orbits of field particles are changed by the interaction with the BH particles as well. Though the field particles do not interact directly with one another, they can indirectly interact through interactions with BH particles.

In § 2, we describe the numerical method we used. In § 3, we describe the result of the simulations with different numbers of particles. Our main result is that the evolution timescale of the binary depends on the total number of field particles, i.e., the ratio between the mass of the BH, $M_{\rm BH}$, and the mass of field particles, $m_{\rm field}$. The relation between t_b and the mass ratio is expressed as

$$t_b \propto \left(\frac{M_{\rm BH}}{m_{\rm field}}\right)^{0.3}, \qquad (2)$$

after the separation of two BHs reached a critical value that depends on the initial structure of the core. If the separation is larger than this critical value, t_b is almost independent of N. This critical value can be qualitatively interpreted as the loss cone radius but is much smaller than the prediction of equation (1). The more sophisticated treatment by Quinlan (1996) seems to give a better explanation. Section 4 contains discussions.

2. NUMERICAL METHOD AND INITIAL CONDITION

2.1. *Numerical Method*

The calculation code we used is NBODY1 (Aarseth 1985) modified to be used with a GRAPE-4 special purpose computer for gravitational N-body simulation (Taiji et al. 1996). The time integration scheme is changed to the Hermite scheme (Makino & Aarseth 1992) to take advantage of the GRAPE-4 hardware. We neglect relativistic effects and treat BH particles as massive Newtonian particles. This treatment is good as long as the periastron distance of the BH pair does not become very small. Our simulations were terminated well before the relativistic effect became important. We used softened potential for the force between field particles, and pure $1/r$ potential for forces to and from BH particles. The relative accuracy of the force calculated on GRAPE-4 is about seven digits (IEEE single precision), which is more than enough for forces from field particles but might not be sufficient for the force from BH particles. In the present code, the force from BH particles is calculated on the host computer in full double precision, in order to avoid possible round-off problems. The force from field particles is calculated on GRAPE-4.

Calculations were performed on one cluster of the GRAPE-4 system with the theoretical peak speed of 270 Gflops. Actual sustained speed was around 100–150 Gflops for simulations with 128K–256K particles. A 256k run took about 5 CPU days.

2.2. *Initial Conditions*

The procedure to prepare the initial condition is the same as described inPaper I. We used the King model with the nondimensional central potential $W_{\rm c} = 7$ as the initial galaxy model. The number of particles is 2048 to 26,2144. The initial galaxy is created so that the total mass M_g is 1 and the total energy E_g is $-\frac{1}{4}$, in the system of units where the gravitational constant G is 1 (the standard unit; see Heggie & Mathieu 1986). The mass of a field particle is $m_{\rm field} = 1/N$. The half-mass radius of the initial galaxy model is about 0.75, and the half-mass crossing time is $2(2)^{1/2}$. To place the central BH, we removed $M_{\rm BH}/m_{\rm field}$ particles closest to the center of the galaxy at time $t = 0$ and put the BH particle at the center. Thus, the galaxy does not initially have a strong central cusp. In Paper I, we found that the structure of the merger does not depend on the details of the procedure to place the BH particles. For all

calculations, $M_{BH} = 1/32$ unless otherwise specified. The softening for the field particles is 1/1024.

The initial orbit of two galaxies is parabolic, with the periastron distance equal to 1. The initial separation of two galaxies is 10. We integrated the system to time $t = 60$.

3. RESULTS

3.1. *The Evolution of the Black Hole Binary.*

Figure 1 shows the time evolution of the energy of the BH binary, E_b, for all runs. The energy E_b is defined as

$$E_b = \frac{1}{2}\mu v_b^2 - \frac{M_{BH}^2}{r_b} = -\frac{M_{BH}^2}{2a}, \quad (3)$$

where $\mu = M_{BH}/2$ is the reduced mass of the BH binary, r_b and v_b are the relative distance and velocity of the two BHs, and a is the semimajor axis of the BH binary.

Before two galaxies merge, each BH lies at the center of its parent galaxy. Thus, the distance is large, and the binding energy of two BHs is negligible. In this period, E_b is roughly equal to the sum of the kinetic energies of two BHs. The energy E_b takes a maximum around $t = 13$ because at that time two galaxies pass the periastron. The relative velocity of two galaxies is largest at the periastron. By $t \simeq 30$, the second fallback takes place and two galaxies merge. Soon after that, two BHs form a binary.

From Figure 1 we can see that the hardening rate of binaries, $-dE_b/dt$, is smaller for larger N. Moreover, it seems that the difference in the hardening rate becomes larger as the binary evolves. Until E_b reaches around -0.05, the growth rate is similar for all runs except for $N = 2048$. In the 2048 body run, one of the BH particles had formed a bound pair with a field particle before it became bound with another BH. When two BHs became bound, this field particle was still bound to a BH particle. This particle was bound to the BH binary until $t = 40$. This is the reason that the curve for the 2048 body run is very noisy.

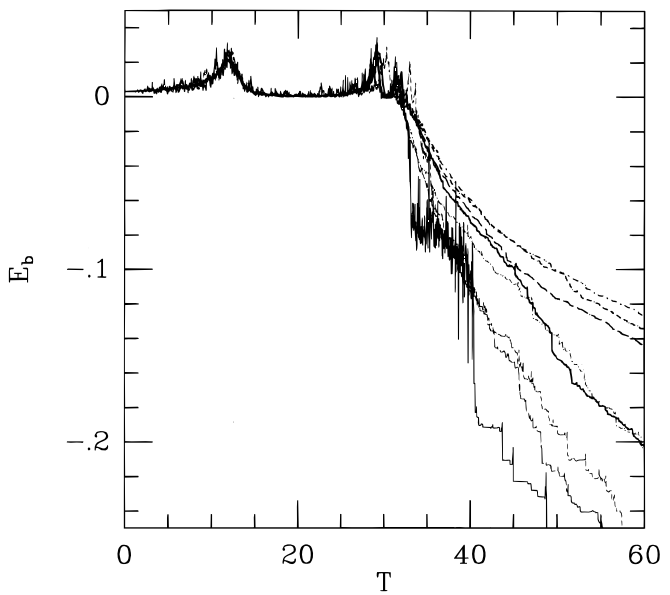

FIG. 1.—The time evolution of the energy of the BH binary for runs with $N = 2$k (*bottom*) to 256k (*top*). Thin lines represent 2k (*solid lines*), 4k (*short-dashed lines*), 8k (*long-dashed lines*), and 16k (*dot-dashed*) runs. Heavy lines represent 32k (*solid lines*), 64k (*short-dashed lines*), 128k (*long-dashed lines*), and 256k (*dot-dashed lines*) runs.

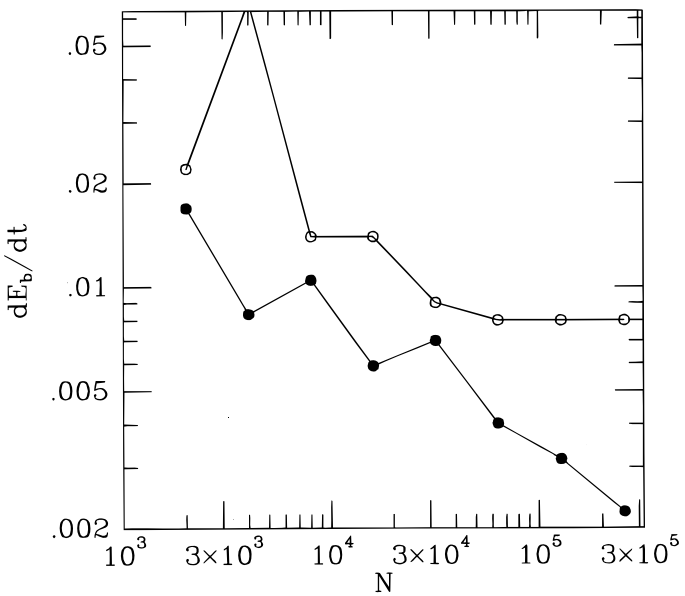

FIG. 2.—The growth rate of the binary binding energy $-dE_b/dt$ plotted vs. the total number of particles N. Open and filled circles are the average rates for $-1/160 > E_b > -1/80$ and $-1/10 > E_b > -1/5$, respectively.

In order to see the dependence of the hardening rate on the energy itself, we plotted $-dE_b/dt$ as a function of N in Figure 2. Here $-dE_b/dt$ is calculated for two intervals of E_b, $(-1/160, -1/80)$ and $(-1/10, -1/5)$. Note that for some runs E_b at the end of the run shown in Figure 1 did not reach $-\frac{1}{5}$. For these runs, we extended simulations so that $|E_b| > 0.2$ at the end of the run.

Here we can clearly see that the hardening rate at the early phase of the evolution seems to converge to $-dE_b/dt \sim 0.008$ for large N, while that for the later phase shows the power-law–like behavior expressed roughly as $dE_b/dt \propto N^{-1/3}$ for the entire range of N.

Figure 3 shows the evolution of E_b for repeated 256k runs. Here the final merger of one run is used as the initial

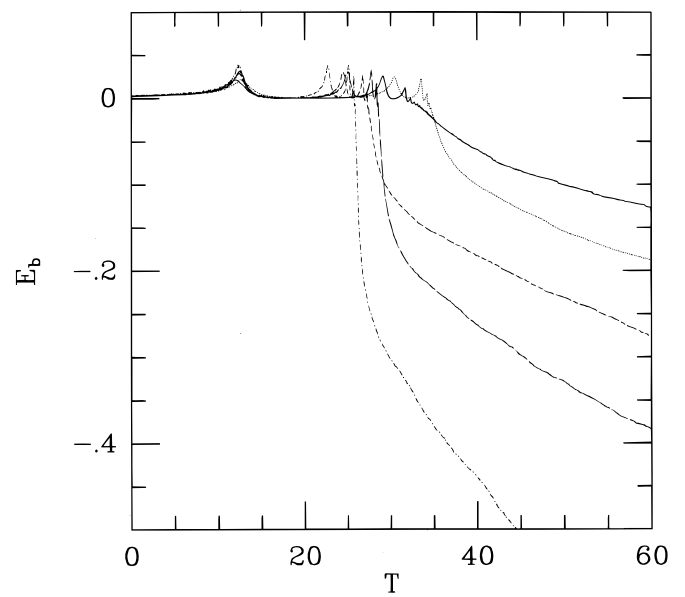

FIG. 3.—Same as Fig. 1, but for repeated mergers from the $N = 256$k run. Solid, dotted, short-dashed, long-dashed, and dot-dashed lines represent first through fifth mergers, respectively.

galaxy of the next run. The initial model is constructed from the merger remnant by the following procedure (see Paper I for details). First, we removed the particles with positive binding energy. Then we replaced the two heavy particles (BHs) by their center of mass and selected half of the field particles with odd indices. Then we scaled the positions, velocities, and mass of all particles so that the half-mass radius of the system becomes 0.75. Each run is terminated at time $T = 60$, which is somewhat arbitrary.

In these runs, the central density of the merger becomes somewhat higher as we repeat the merging process (see Fig. 4*e* for the density profiles). Since the timescale of the evolution of the binary is proportional to the central density, this difference in the central density resulted in the difference in the growth timescale. This difference in the central density also resulted in the difference in the critical value of E_b.

There is no reason to assume that the first merger would give a reasonable result for the timescale, since our choice of the initial model is rather arbitrary. The structure of the core in the last merger events would be a more appropriate model for the initial galaxy. In this case, the critical value for E_b is around -0.3.

3.2. *The Structure of the Core*

Figure 4 shows the density profiles of the central region of the merger remnant from 256k, 64k, and 16k runs for different times. For the 256K run, the density is almost flat at the very central region, with possible minimum at $r \sim 0.01$, where r is the radius. The density decreasing inward is also visible in Figure 5, where the result of five repeated mergings with 256k particles is plotted.

The difference in the central structure explains the differ-

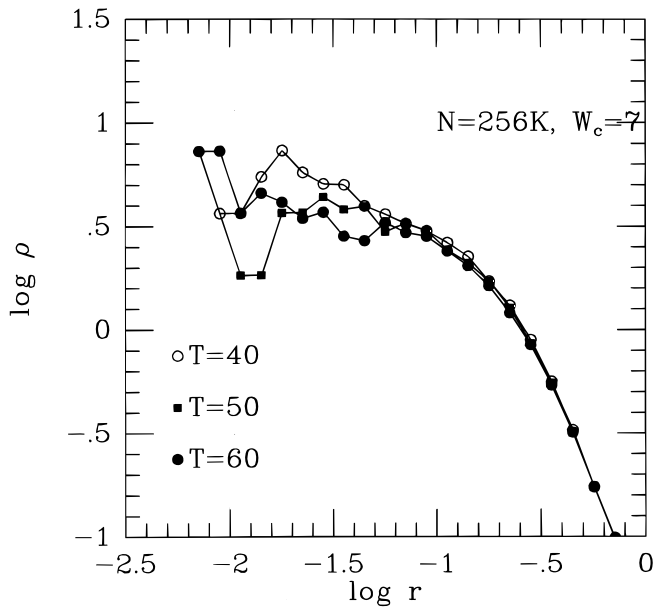

FIG. 4*a*

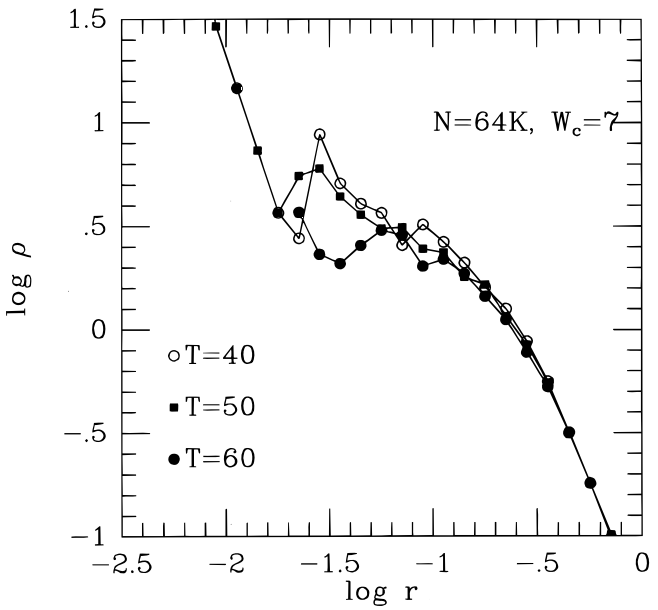

FIG. 4*b*

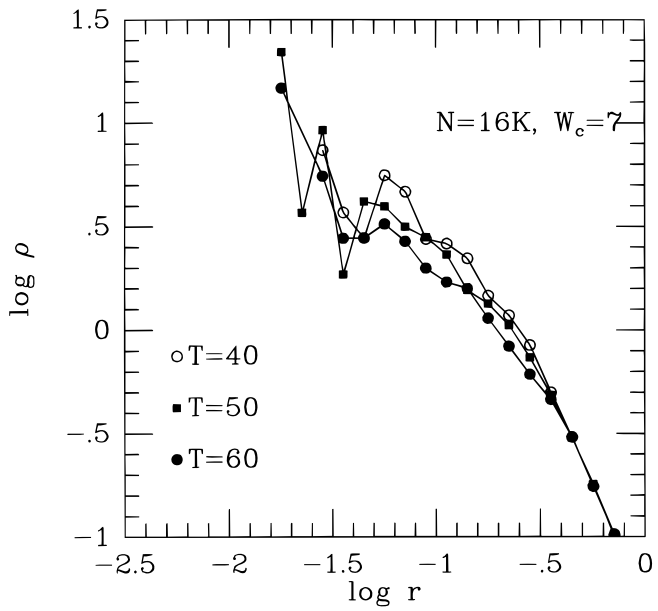

FIG. 4*c*

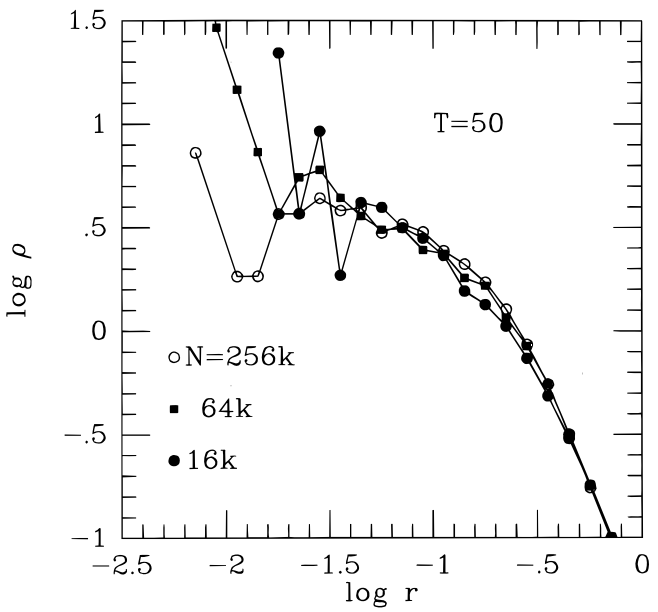

FIG. 4*d*

FIG. 4.—Density profiles of the central region of the mergers: runs with (*a*) 256k, (*b*) 64k, and (*c*) 16k particles. Open circles, filled squares, and filled circles are the profiles at $t = 40$, 50, and 60, respectively. Figure 4*d* shows the profiles at $t = 60$ for different total number of particles.

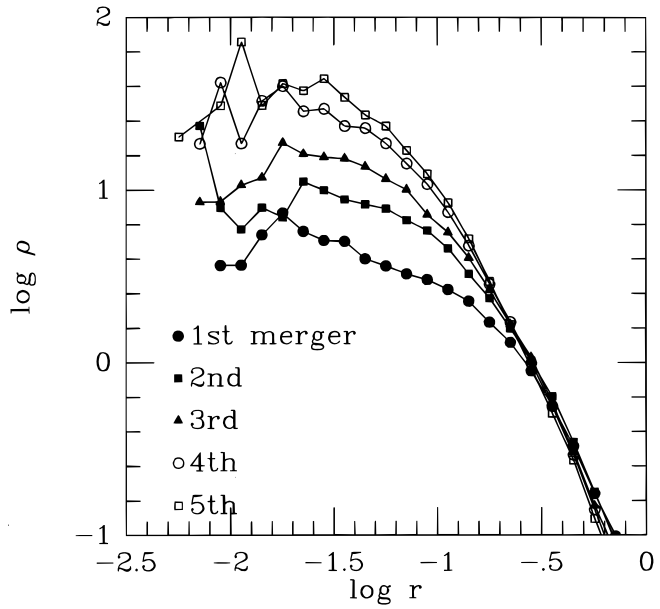

FIG. 5.—Density profiles of repeated mergings. Filled circles, filled squares, filled triangles, open circles and open squares represent the profiles for first through fifth mergers. For all profiles, $t = 40$. Initial total number of particles is 256k.

ence in the growth rate of E_b shown in Figures 1 and 2. The growth rate is proportional to the stellar mass density around the BH binary. Figure 4*d* suggests that the stellar density around the BH binary is lower for simulations with larger N. If we compare the profiles for 64k and 16k runs, the stellar density at the innermost shell is slightly larger for the 64k run; this is essentially because of the small number of particles in the innermost shells. In fact, at $t = 60$ central densities for 64k and 256k runs are about the same, while that for the 16k run is much higher. In Figure 5, central density for most runs shows the tendency to decrease inward, which might indicate the loss cone itself.

The difference in the structure of the core is due to the difference in the central relaxation time. Since the merger remnant does not have a flat core, the central relaxation time is rather difficult to define. We calculated the local relaxation time as the function of the radius r, and we found that it takes the minimum value at $r \sim 0.1$ for all models. We regard this minimum value as the central relaxation time, t_{rc}. At time $t = 40$, t_{rc} is around 400, 1200, and 5000 for 16k, 64k, and 256k runs, respectively. Thus, for the 16k run the simulation time span is still a sizable fraction of t_{rc}, while for the 256k run t_{rc} is orders of magnitudes larger than the simulation time span.

The flat central region observed in the 256k run is a kind of the loss cone predicted by BBR. In the theory of BBR, when the binary becomes sufficiently hard, it kicks out all the stars it can interact with, and then the growth slows down significantly.

However, it should be noted that the structure we observed is rather different from the picture of BBR or the standard model for the star cluster with massive central BH (Shapiro 1985, and references therein), which predict the cusp with $\rho \propto r^{-1.75}$ outside a few times the radius of the BH binary. For the 256k run, the numerical result is a very shallow cusp outside the loss cone, something like $\rho \propto r^{-0.5}$. For runs with smaller N the slope is steeper. Thus, it seems that the cusp becomes shallower as we increase N. On the other hand, the theoretical prediction is a universal cusp of $\rho \propto r^{-1.75}$. We will discuss possible interpretations in § 4.

Figure 6 gives the surface brightness profiles for same runs as in Figure 4. Here the time evolution is not as clear as that in Figure 4. In addition, we cannot see any decrease of the luminosity toward the center, even for the 256k run.

4. SUMMARY AND DISCUSSION

We performed the simulation of the evolution of massive BH binaries formed through mergings of elliptical galaxies by means of direct N-body simulations. Our major findings are summarized as follows.

1. The timescale of the evolution of the binary depends on the total number of particles, in other words, the mass ratio between BHs and field particles. However, the dependence is very weak for the early phase of evolution. For the later phase, we observed $t_b \propto N^{1/3}$, which is much weaker than the prediction of the theoretical model.
2. The evolution timescale shows very strong dependence on the initial central density.
3. For large N runs, the "loss cone" depletion effect is clearly visible. In other words, the stellar density around the BH is lower for larger N. However, again, the density profile we obtained for simulations with large N was quite different from the theoretical model, which gives the density cusp of $\rho \propto r^{-1.75}$.

In the following, we discuss the implication of these results in some detail.

4.1. *The Hardening Rate of the Black Hole Binary*

For the early phase of the evolution, it is not surprising that the evolution timescale is independent of N. This is because the loss cone depletion has not taken place. However, our result shows that the critical semimajor axis of the binary at which the loss cone effect becomes significant is much smaller than the prediction by BBR.

This difference might be due to the assumption of BBR in calculating the loss cone radius. They assumed that the loss cone depletion occurs when the total kinetic energy of all stars that can interact with the binary becomes smaller than the binding energy of the binary. Quinlan (1996) argued that the loss cone depletion occurs only when all particles that can interact with the BH binary are ejected out of the core. In his examples, this assumption leads to a loss cone radius a few times smaller than the semimajor axis for the "1 kT" binary (the binary with the orbital velocity comparable to the typical velocity of field stars). His result is qualitatively consistent with our present result. For our runs, the critical value of E_b is $-2 \sim 10 M_{BH} v_c^2$, depending on the density profile, where v_c is the rms velocity of field particles.

When the BH binary reached the above critical point, its evolution slowed down. We found that the dependence of the growth timescale on N is much weaker than the theoretical prediction of $t_b \propto N$. This difference might be understood by means of the detailed treatment of the evolution of the stellar distribution around the central BH developed by Shapiro and his collaborators (Shapiro 1985; Duncan & Shapiro 1983).

The essence of their argument is that the stars are removed from the system only when they actually come close enough to the BH (they considered the distribution of stars around a single BH). If the average change of the

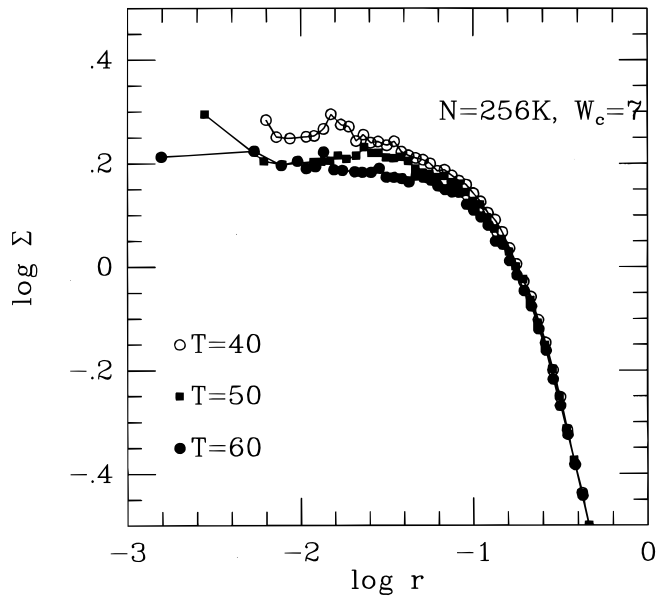

FIG. 6*a*

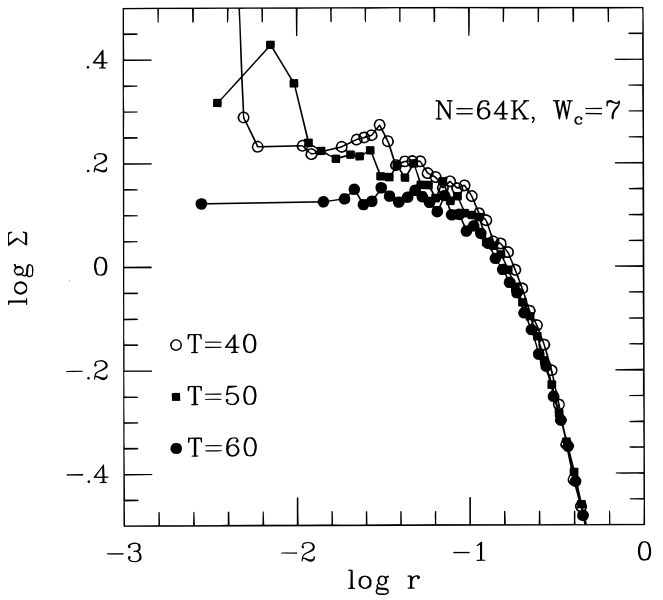

FIG. 6*b*

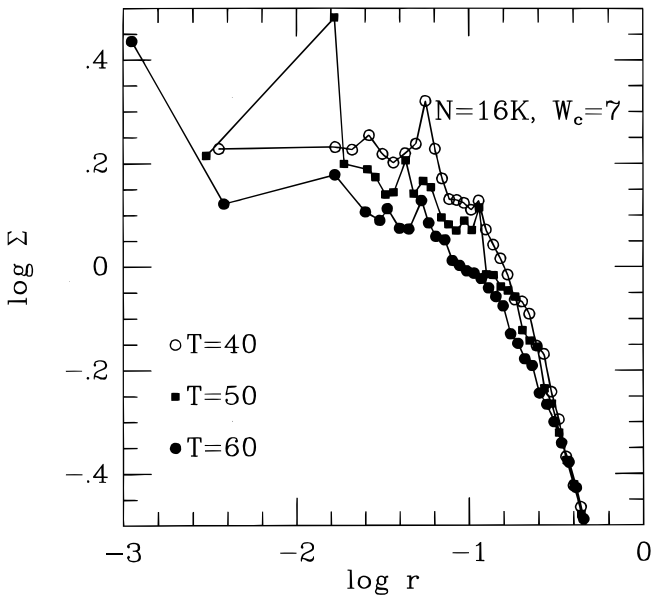

FIG. 6*c*

FIG. 6.—Same as Fig. 4, but for surface density profiles

angular momentum of a typical star in the core in one orbital period is larger than the maximum angular momentum of the star in the loss cone orbit, the loss cone is repopulated in the timescale shorter than the depletion timescale. In this case, the dependence of the growth timescale on N might be weaker than the theoretical prediction.

The maximum angular momentum of a field particle, $J_{\max}$, that can interact with the BH binary is expressed roughly as

$$J_{\max} \sim \sqrt{M_{\rm BH} a} \sim 0.01 E_b^{-1/2} , \tag{4}$$

where a is the semimajor axis of the BH binary, and we used $M_{\rm BH} = 1/32$.

On the other hand, the average change in the angular momentum of a typical star in the core per orbit is given roughly by

$$\Delta J \sim r_c v_c \sqrt{t_c / t_{\rm rc}} , \tag{5}$$

where r_c, v_c, t_c, and $t_{\rm rc}$ are the core radius, the rms velocity of the particles in the core, the core crossing time and the core relaxation time, respectively. If we use $r_c = 0.1$ and $N_c = N/20$ as a typical value for our simulation (see Fig. 4), we have

$$\Delta J \sim \sqrt{\frac{2 \log (0.02N)}{N}} . \tag{6}$$

For $E_b \sim -0.1$, $\Delta J > J_{\max}$ in the case of $N = 2048$ and the opposite in the case of $N = 262144$. Thus, the range of N in our simulation just covers the transition from $J_{\max} < \Delta J$ to $J_{\max} > \Delta J$. It is not at all surprising that the dependence of the growth rate is weaker than the prediction of the theory, which assumes $J_{\max} \gg \Delta J$.

There are two other mechanisms that might affect the evolution timescale of the binary. As we mentioned in § 1, the random motion of the BH binary might play some role,

since it (1) increases the loss cone radius and (2) increases the timescale of the loss cone depletion. Figure 7 shows the dependence of the rms position and velocity of the center of mass of the BH binary. These are calculated for $50 \geq t \geq 60$. Both quantities are proportional to $N^{-1/2}$. In our simulations, the semimajor axis at the critical separation is about 0.005. Thus, the distance that the BH binary wanders around is larger than the size of the binary for all values of N. Simulations with $N > 10^6$ are necessary to suppress this effect.

Very recently, Rauch & Tremaine (1996) proposed a new mechanism of relaxation, which they termed "resonant relaxation." If the particles orbit under "almost Keplerian" potential, they change the orbit very slowly. Therefore, the normal two-body relaxation is enhanced by a factor proportional to the inverse of the precession rate of the orbit. They discussed the possible effect of this resonant relaxation to the evolution timescale of the BH binary and concluded that it had a rather minor effect, assuming the presence of the $-7/4$ cusp around the BH binary. However, their treatment is highly simplified and needs to be tested with more detailed calculations.

In order to determine really the dependence of the lifetime on N, we need to employ more particles. Simulations with numbers of particles a few times 10^6 would be necessary.

Simulations with numbers of particles larger than those used here might be possible by means of more approximate schemes, such as the tree code with individual time steps (McMillan & Aarseth 1993) or an extension of the self-consistent field method (see Quinlan, Hernquist, & Sigurdsson 1995) to the case of the central BH binary.

It should be noted that here we study the *thermal* evolution of the system, since if the system is collisionless, then the loss cone will never be repopulated. Thus, the two-body relaxation effect should be modeled appropriately. The use of so-called collisionless schemes should be done very carefully.

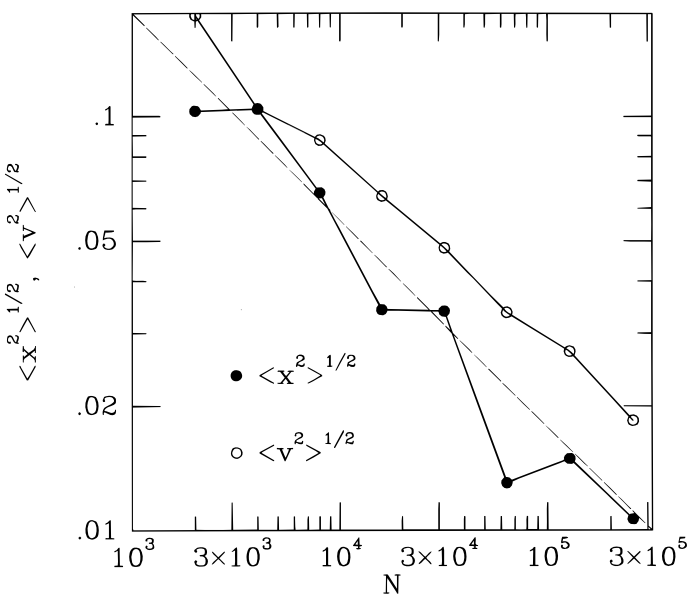

FIG. 7.—The rms position (*filled circles*) and velocity (*open circles*) of the center of the BH binary plotted against the number of particles. The dashed line indicates $N^{-1/2}$.

4.2. *The Evolution of Black Hole Binaries in Real Elliptical Galaxies*

Here we discuss the evolution of BH binaries in real elliptical galaxies. For simplicity, we assume that the evolution of the binary stops at the critical energy obtained in our experiments.

The timescale of the gravitational radiation is given by

$$t_{\rm GR} = 2 \times 10^{15} g(e) (M_{\rm BH}/10^8\ M_\odot)^{-3} (a/1\ {\rm pc})^4\ {\rm yr}\ , \quad (7)$$

with $g(e)$ given by

$$g(e) = \frac{(1-e^2)^{7/2}}{1 + (73/24)e^2 + (37/96)e^4} \simeq 2(1-e)^{7/2} (e \sim 1)\ , \quad (8)$$

which, in our units,

$$t_{\rm GR} = 5 \times 10^9 g(e) (M_{\rm BH}/10^8\ M_\odot) (v_c/300\ {\rm km\ s^{-1}})^{-8} E_b^{-4}\ {\rm yr}\ . \quad (9)$$

Thus, the lifetime of the BH binary with $E_b = -0.05$ is $\sim 10^{15}$ yr, which is well over the Hubble time. For $E_b = -0.3$, the lifetime is more than 10^{11} yr. Unless the eccentricity is relatively large, it is unlikely for the BH binary to merge.

If the lifetime of the BH binary is long, many merger remnants might have central BH binaries. BBR (see also Rees 1990) argued that such a BH binary is likely to be kicked out from the core or the galaxy itself through the gravitational slingshot when the parent galaxy merges with yet another galaxy.

However, the slingshot mechanism is not as effective as assumed by BBR. When one BH binary and a single BH reside in the core, there are two possible outcomes. In one case, at least one of the three BHs is ejected out of the parent galaxy through the slingshot. In the other case, the binary merges through gravitational radiation before kicking out the third BH. BBR concluded that the slingshot is the likely outcome, assuming that the binary is circular. Makino & Ebisuzaki (1995) showed that the slingshot is unlikely to occur if the distribution of the eccentricity is correctly taken into account. They showed that the typical lifetime of the BH binary before the ejection of the third body is less than 10^9 yr. If the BH binary merged before the third BH is ejected out of the galaxy, the final state is the binary of the merged BH and the third BH.

Thus, it is quite possible that a large fraction of elliptical galaxies have central BH binaries. The implication of the presence of the binary was addressed briefly in BBR. The fate of the BH binary depends most strongly on v_c. If $v_c < 200$ km s^{-1}, the mechanism described in Makino & Ebisuzaki (1995) cannot make the BH binary merge before it kicks out the third body. On the other hand, if $v_c > 600$ km s^{-1}, then the BH binary would merge through the dynamical friction and the gravitational radiation. Since most elliptical galaxies fall in the range of 200 km s^{-1} $< v_c <$ 600 km s^{-1}, the fate of the BH binary is difficult to predict. Interaction with gas might determine the final fate.

If triple BH systems are not very uncommon, then we might be able to observe some of them as the QSO activities located far from the center of the galaxy. Recent *Hubble Space Telescope* (*HST*) observations of nearby QSOs (Bahcall, Kirhakos, & Schneider 1995a, 1995b, 1995c) sug-

gested that many of the QSOs are not located at the center of the galaxy. In the triple BH system, BHs would spend most of the time in the halo "parking" orbit, as in the case of the binaries in globular clusters (Hut, McMillan, & Romani 1992). Thus, QSO activities might be observed in the positions far from the center of the parent galaxy. To explain QSOs outside the core of the parent galaxy, Fukugita & Turner (1996) proposed a scenario in which the massive BHs are formed independently from the galaxies. Such an exotic model might not be necessary if triple BHs are common.

4.3. *The Structure of the Central Region*

Our numerical result for the density profile around the BH pair is quite different from the standard picture of the cusp with $\rho \propto r^{-7/4}$. Though the cusp exists, its slope is smaller for larger N.

For large-N runs, it is quite natural that the slope of the cusp is different from that of the theoretical prediction, simply because the duration of the simulation is much shorter than the two-body relaxation time of the core. It takes the thermal relaxation time of the core for the cusp to fully develop. For the 256k run, the local two-body relaxation time at $r = 0.1$ is 5×10^3 for $t = 40$. Thus, the cusp cannot develop fast enough.

Since the two-body relaxation has an even smaller effect in real elliptical galaxies than in our largest simulations, we can safely predict that the $r^{-7/4}$ cusp is not present in elliptical galaxies with a central massive BH. The structure is most likely to be a cusp of $r^{-0.5}$ formed by the particles not bound to the BH binary (Duncan & Shapiro 1983), terminated at the loss cone radius. This is in good agreement with the resent *HST* observations of the core of large elliptical galaxies, which show the cusp with a slope of $-0.5 \sim -1$. This shallow cusp indicates that the central BH is placed in the galaxy in a dynamical timescale. If the BH mass grows in the timescale longer than the dynamical time, the slope would be -1.5 (Young 1980).

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