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TRIPLE BLACK HOLES IN THE CORES OF GALAXIES

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ABSTRACT

We investigate the evolution of black hole (BH) triples in the cores of galaxies. A triple BH system can be formed when a merger remnant of ellipticals having central BHs merges again with another galaxy. The spiral-in of massive BHs, which are proposed as a candidates for constituents of the dark halo of our Galaxy, can also make a triple BH system in the core of the Galaxy. It has been argued that such a triple system will inevitably result in the ejection of at least one of the BHs by the gravitational slingshot. We will show that such a slingshot is unlikely to occur and that two of the three BHs are likely to merge by gravitational radiation. The final state is the binary of this merged BH and the third BH, which may or may not merge by the combined effect of dynamical friction from the stars in the core and gravitational radiation.

Subject headings: black hole physics — celestial mechanics, stellar dynamics — galaxies: interactions — galaxies: nuclei

1. INTRODUCTION

The evolution of black hole (BH) binaries in the cores of galaxies has attracted the attention of many researchers. Begelman, Blandford, & Rees (1980, hereafter BBR) pointed out that such a binary might be the cause of the observed bending and apparent precession of radio jets from active galactic nuclei (AGNs). In their scenario, a binary BH is formed when two parent galaxies, both having massive BHs, merge to form one galaxy. After the two galaxies have merged, the two BHs sink into the core of the merger remnant due to dynamical friction and form a binary system.

After the binary is formed, it evolves further through interaction with field stars in the core. On average, field stars gain kinetic energy through interaction with the binary BH. As a result, the semimajor axis of the binary shrinks and the binary becomes harder. As long as there are stars in the neighborhood of the BH binary, it continues to harden. When the separation becomes small enough, the gravitational wave radiation becomes important and the binary will merge.

BBR predicted that the timescale of the merging was much longer than the Hubble time. Their argument can be summarized as follows: if the binary is sufficiently hard, the field stars that interact with the binary will be ejected from the core of the galaxy. Thus, the binary will soon run out of stars with which it can interact. Further evolution of the BH binary will proceed on the timescale on which the field stars diffuse into the "loss cone" formed around the BH binary. Once the loss cone is created, the evolution timescale of the BH binary will be much larger than the Hubble time. Therefore, the BH binary is likely to remain unmerged when another BH sinks into the core as the result of merging with another galaxy. BBR argued that the third BH (and in some cases the BH binary itself) would be ejected from the galaxy as the result of the three-body interaction.

Ebisuzaki, Makino, & Okumura (1991) performed self-consistent N-body simulations of the merging of spherical galaxies containing BHs. They found that energy production from the BH binary plays the primary role in determining the core radius of the merger remnant. In numerical simulations of mergers without central BHs, the core size of the merger turned out to be nearly equal to the core size of the progenitors, while the effective radius became about 2 times larger (e.g., Farouki, Shapiro, & Duncan 1983; Okumura, Makino, & Ebisuzaki 1991). This result is inconsistent with observations, which show a positive correlation between the core radius and the effective radius (e.g., Lauer 1985).

In the merging of galaxies with central BHs, the core of the merger expands due to energy generation from the BH binary. The ratio between the core radius and the effective radius is roughly proportional to the ratio between the BH mass and the mass within the effective radius. Numerical simulations by Ebisuzaki et al. (1991) and more detailed simulations by Makino & Ebisuzaki (1994a) successfully reproduced the observed linear relation between the core radius and the effective radius.

Both Ebisuzaki et al. (1991) and Makino & Ebisuzaki (1994a) assumed that the BH binary merges before the merger remnant merges again with a third galaxy. As discussed by BBR, whether this assumption is valid is unclear. If the loss cone is actually formed as suggested by BBR, the timescale of merging would be longer than the Hubble time. It would be difficult to explain the linear relationship between the core radius and the effective radius by the energy input from BH binary, since it seems unlikely that the BH mass grows in proportion to the galaxy mass. When the BH binary interacts with another BH (or BH binary), at least one of the BHs would be ejected.

Mikkola & Valtonen (1992) studied the evolution of the BH binary in the core of a galaxy using a restricted N-body method, in which the interaction between the field stars is approximated as the force from a fixed potential field. They found that the binary continued to harden beyond the limit predicted by BBR. Their results might have suffered from the small-number effect, since the mass ratio between the BH and field stars was only 100 in their simulations. However, Quinlan & Hernquist (1994) performed similar calculations with a much larger number of particles and obtained a similar result. Makino & Ebisuzaki (1994b) performed self-consistent

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N-body simulations of mergers with central BHs with mass ratios up to 1024. They also found that the BH binary evolves far beyond the limit given by BBR. In addition, they found that the eccentricity of the BH binary was likely to be large for realistic initial conditions. They concluded that the lifetime of the BH binary would be 10^6–10^9 yr, depending on the eccentricity. In their simulations, however, the final eccentricity of the BH binary exhibited large fluctuations. Therefore, it is not rare that the BH binary would survive for 10^9 yr or longer.

In this paper, we investigate what will happen if there are three BHs (one binary and a third one) in the core of a galaxy. BBR assumed that the three-body interaction would cause at least one of the three BHs to be ejected from the galaxy. This assumption is, however, not true if we take into account the effect of gravitational radiation. The eccentricity of the binary after a binary-single BH encounter is expected to have the thermal equilibrium distribution of $f(e) = 2e$ (e.g., Heggie 1975). The removal of one BH typically requires several such encounters, unless the initial BH is already much harder than the predicted "loss-cone" limit of BBR. As a result, the maximum eccentricity that the BH binary experiences is significantly larger than the average value of 0.7. For example, if the BH binary experienced 10 encounters, the median value of the maximum eccentricity is as large as 0.97. A binary with an eccentricity of 0.97 has a lifetime roughly 10^5 times shorter than a circular binary with the same semimajor axis, since the timescale of the gravitational radiation is proportional to $(1 - e)^{-3.5}$. In the next section, we estimate the lifetime of a binary BH, taking into account this effect. In § 3, we apply the result obtained in § 2 to the case of the massive ($\sim 10^8\ M_\odot$) BH binaries in the core of an elliptical galaxy. We found that the typical timescale of gravitational radiation is about 10^8 yr, which is comparable to the timescale of the fallback of the BH from the recoil. In other words, the BH binary is likely to merge before it ejects the third body. In § 4, we consider the case of a BH of mass $10^6\ M_\odot$. Lacey & Ostriker (1985) proposed such massive BHs as the main constituent of the dark halo of our Galaxy. If such massive BHs exist in the halo, many of them would have sunk to the Galactic center due to dynamical friction. Lacy & Ostriker argued that the mass of the BHs in the Galactic center would not grow, since three-body interaction would remove BHs from the core. We show that the probability of merging due to gravitational radiation is unacceptably high if the BH mass is larger than $3 \times 10^4\ M_\odot$.

2. THE LIFETIME OF A BH BINARY

Here, we evaluate the lifetime of a BH binary as a function of the escape velocity v_e of the galaxy and the mass of the black hole $M_{\rm BH}$. The evolution of the triple system is described by the repeated binary-single BH scattering events. If the recoil velocity of the single BH is less than the escape velocity, it will soon fall back and interact with the binary. If the lifetime of the BH binary is shorter than the fallback time, t_f, the BH binary will merge before the next scattering. In this case, the final state is a binary of the merger BH and the third one.

The probability that the BH binary merges before the ejection of the third BH from the galaxy is expressed as

$$P(t) = 1 - \prod_{k=0}^{n} P_k(t_f)\,, \tag{1}$$

where n is the number of scattering events before the ejection of the third body, t_f is the typical fallback time, and $P_k(t)$ is the probability that the lifetime after scattering event k is longer that t. Here, we number the scattering events backward in time, calling the event that would result in the escaping of one BH event 0.

The probability, $P_i(t)$, is estimated as follows: The timescale of the orbital decay of a BH binary, with eccentricity e and semimajor axis a, due to gravitational radiation is expressed as (Peters 1964)

$$t_{\rm GR} = g(e)t_{\rm GR,0}\,, \tag{2}$$

where $t_{\rm GR,0}$ is the lifetime of a circular binary ($e = 0$) with semimajor axis a and $g(e)$ is a function of the eccentricity, e. These are expressed as

$$t_{\rm GR,0} = 2 \times 10^{15}(M_{\rm BH}/10^8\ M_\odot)^{-3}(a/1\ {\rm pc})^4\ {\rm yr} \tag{3}$$

and

$$g(e) = \frac{(1 - e^2)^{7/2}}{1 + (73/24)e^2 + (37/96)e^4} \simeq 2(1 - e)^{7/2} \quad (e \sim 1)\,. \tag{4}$$

Equation (2) states that $t_{\rm GR}$ is larger than a given time t if $g(e) > t/t_{\rm GR,0}$. $P_k(t)$ is expressed as

$$P_k(t) = \int_{e=0}^{e=e_{\rm max}} f(e)de = e_{\rm max}^2\,, \tag{5}$$

where $e_{\rm max}$ is the eccentricity for which $g(e) = t/t_k$. We used the thermal distribution of the eccentricity $f(e) = 2e$ (Heggie 1975). Note that $g(e)$ is a monotonically decreasing function of e.

Using equation (4), $e_{\rm max}$ is expressed as

$$e_{\rm max} = 1 - \left(\frac{t}{2t_k}\right)^{2/7}, \tag{6}$$

where t_k is the lifetime of the binary after scattering event k for a circular orbit, Therefore, $P_k(t)$ is expressed as

$$P_k(t) = 1 - 2\left(\frac{t}{2t_k}\right)^{2/7}, \tag{7}$$

if $t \ll t_k$.

The lifetime for the circular orbit t_k is calculated as follows: The binding energy increases, on average, by 40% at each scattering event (see, e.g., Heggie 1975). The semimajor axis after event k is given by $a_k = (1.4)^k a_e$, where a_e is the semimajor axis of the binary which has just enough binding energy to eject one BH in the next scattering. From equation (3), t_k is expressed as

$$t_k = t_e\left(\frac{a_k}{a_e}\right)^4 = (1.4)^{4k}t_e \simeq 4^k t_e\,, \tag{8}$$

where t_e is the lifetime of the circular binary with semimajor axis a_e. From equations (1), (7), and (8), the probability that the BH binary merges before the ejection of one BH is given by

$$P(t_f) = 1 - \prod_{i=0}^{n}\left[1 - 2\left(\frac{4^{-i}t_f}{2t_e}\right)^{2/7}\right] = 1 - \prod_{i=0}^{n}\left[1 - 1.64(0.672)^i\alpha^{2/7}\right], \tag{9}$$

where $\alpha = t_f/t_e$. Note that α gives the typical interval of the scattering events measured in units of the lifetime of a circular

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binary. In other words, α gives the factor by which the estimate, assuming a circular orbit, overestimates the lifetime. Assuming that $\alpha \ll 1$ and that $n \gtrsim 5$, we obtain

$$P(t) \simeq 1.64\alpha^{2/7} \sum_{i=0}^{n} (0.672)^i \simeq 5\alpha^{2/7} . \quad (10)$$

The value of α for $P = 0.5$ is 0.3×10^{-3}. In other words, the typical lifetime of the BH binary is shorter than the estimate for the circular binary by a factor of 3×10^3.

Finally, we estimate t_e. We consider the escape of the third body and the escape of the binary itself, separately. In the case of the massive BHs in merger remnants, the probability of merging before the ejection of one BH is important. For the case of BHs as dark-halo constituents, the probability of merging before the ejection of the binary is important. The average energy increase by one binary-single BH interaction is about 40% (Heggie 1975). Roughly two-thirds of this energy is transferred to the kinetic energy of the single body. If the escape velocity of the galaxy at the core is v_e, the separation, a_e, of the binary before ejecting the third body is determined to be

$$\frac{1}{2} v_e^2 = 0.4 \times \frac{2}{3} \times \frac{GM_{\rm BH}}{2a_{e,s}} \quad (11)$$

for the single BH and

$$a_{e,b} = \frac{a_{e,s}}{4} \quad (12)$$

for the binary.

Equation (11) can be rewritten as

$$a_{e,s} = 0.23(M/10^8\ M_\odot)(v_e/1000\ \text{km s}^{-1})^{-2} . \quad (13)$$

From equations (2) and (13), we have

$$t_e = \begin{cases} 6 \times 10^{12}(M_{\rm BH}/10^8\ M_\odot)(v_e/1000\ \text{km s}^{-1})^{-8}\ \text{yr} \\ \qquad\qquad \text{(single BH)} , \\ 3 \times 10^{10}(M_{\rm BH}/10^8\ M_\odot)(v_e/1000\ \text{km s}^{-1})^{-8}\ \text{yr} \\ \qquad\qquad \text{(binary)} . \end{cases} \quad (14)$$

3. ELLIPTICAL GALAXIES WITH MASSIVE BHs

Table 1 summarizes the parameters we assumed for elliptical galaxies. We assumed that the escape velocity of the galaxy is 4 times larger than the velocity dispersion in the core, which is about 300 km s^{-1}. We consider the lifetime before the ejection of the single BH, since we are interested in whether all of the BH mass is retained in the core. The lifetimes of 50% of BH binaries are shorter than 4×10^8 yr. The typical timescale of the fallback of a single BH ejected from the core of the merger remnant would be of the order of 10^8 yr. Therefore, we can expect that a large fraction of BH binaries would merge before ejecting the third BH. In other words, even if the loss-cone depletion is very effective in extending the lifetime of the BH binary, the mass of the central BH is likely to grow when a merger remnant merges with another galaxy.

TABLE 1

CORE PARAMETERS AND EXPECTED LIFETIME

Parameter	Ellipticals	Our Galaxy
Escaper	Single BH	Binary
v_e (km s^{-1})	1200	400
$M_{\rm BH}$ ($M_\odot$)	10^8	10^6
$t_{\rm GR}$ (circular; yr)	1.3×10^{12}	3×10^{11}
$t_{\rm GR}$ ($P = 0.5$; yr)	4×10^8	1×10^8

It must be noted that our result is extremely sensitive to v_e, since the lifetime is inversely proportional to v_e^8. Therefore, for larger ellipticals, the probability of the merging of central BHs is much higher, since large ellipticals typically have deeper potential.

4. OUR GALAXY

If the dark halo of our Galaxy is composed of BHs with mass $10^6\ M_\odot$, one BH would spiral in to the Galactic center every 10^8 yr (Lacey & Ostriker 1985). In order to prevent excessive mass accretion at the center, the BH binary itself must be ejected from the core. In this case, the typical lifetime is about 10^8 yr. Thus, once formed, the BH binary is likely to merge before being ejected from the core. If one merging has occurred, it becomes very hard to eject the most massive BH from the core (Hut & Rees 1992). Therefore, we can conclude that most of the BHs that have sunk to the core would have merged to form a single massive BH. This result clearly contradicts the observational upper limit for the mass of the central massive object of $3 \times 10^6\ M_\odot$ (Spaenhauer, Jones, & Whitford 1992).

In the following, we give the upper limit for the halo BH mass. As shown in equation (14), the lifetime of the BH binary is proportional to the mass of the BHs, since the dependence of the escape velocity on the BH mass is very weak. Thus, we can conclude that if the BH mass is smaller than $10^6\ M_\odot$, the accumulation of the BH mass at the Galactic center is more likely to occur.

The mass that can fall in to the Galactic center in the Hubble time is proportional to $M_{\rm BH}^{1.5}$, if we assume that the core radius of the initial distribution of BHs is larger than a few kiloparsecs (Lacey & Ostriker 1985). Therefore, in order to prevent the accumulation of a BH mass of more than $10^6\ M_\odot$, the mass of BHs in the halo must be less than $3 \times 10^4\ M_\odot$. This mass is too small to explain, for example, the random velocity of disk stars (Lacey & Ostriker 1985).

5. CONCLUSION

We estimated the lifetime of a triple BH system in the center of a galaxy. Our main conclusion is that the timescale of merging by gravitational radiation is shorter than the simple estimate for a circular binary by a factor of 3×10^3 for 50% of the binaries. This large factor comes from the following two reasons: First, the lifetime depends strongly on the eccentricity. Second, the maximum eccentricity that the binary can reach in several consecutive encounters is much larger than the average eccentricity of 0.7.

For a massive BH in the core of an elliptical galaxy, this effect of the eccentricity reduces the lifetime of the binary to the order of 10^8 yr. In other words, such a binary would merge before it ejects the third body. For BHs in the halo of our Galaxy, the accumulation of the central BH mass is unavoidable, and in order to reduce the central mass below the observational limit, the BH mass must be less than $3 \times 10^4\ M_\odot$.

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