THE ASTROPHYSICAL JOURNAL, 424:L17–L20, 1994 March 20
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ATTENUATION OF LUMINOSITY OF DISTANT GALAXIES BY MULTIPLE GRAVITATIONAL LENSING

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Received 1993 August 13; accepted 1993 November 12

ABSTRACT

We investigated the change of luminosity of distant galaxies due to multiple gravitational lensing. We obtained the luminosity change by calculating the orbits of photons in a model universe in which lensing objects are uniformly distributed. We found that luminosities of galaxies are, in most cases, attenuated by multiple gravitational lensing. There are a few galaxies whose luminosity is amplified through multiple lensing. The number of attenuated galaxies is much larger than that of amplified galaxies. We also found that the median value of attenuation ratio of the distant galaxies is well approximated by $(1+z)^{-1.3(\Omega_l)^{1/2}}$, where Ω_l is the density parameter of lensing objects. This result shows that the gravitational lensing introduces an "attenuation bias" into the luminosity function of distant galaxies and quasars.

Subject heading: gravitational lensing

1. INTRODUCTION

The light from distant sources, such as galaxies and QSOs, experiences deflections due to gravitational field of galaxies, clusters of galaxies, and superclusters during its travel toward us. This deflection is known as gravitational lensing. Both luminosity and shape of a galaxy are changed by the deflection due to gravitational lensing (Schneider, Ehlers, & Falco 1992; Blandford & Narayan 1992).

The images distorted by a single lensing are observed as a ring, an arc or multiple images, depending on the geometry of the lens system (e.g., Lynds & Petrosian 1986; Soucail et al. 1987; Hewitt et al. 1988; Walsh, Carswell, & Weymann 1979; Huchra et al. 1985). In these cases, the luminosity of the source is usually amplified. Some researchers argued that QSOs, BL Lac objects, and galaxy-QSO correlations are produced by this amplification (e.g., Ostriker & Vietri 1990; Zwicky 1937; Press & Gunn 1973; Arp & Hazard 1980; Tyson 1986).

Vietri & Ostriker (1983) investigated the effect of the amplification due to gravitational lensing on the luminosity function of galaxies. In subsequent papers (Vietri 1985; Ostriker & Vietri 1986), they concluded that the statistical effect of the gravitational lensing was not strong enough to affect the luminosity function. They assumed that the deflection due to multiple lensing is expressed by linear superposition of deflections due to individual objects (Gunn 1967). This assumption, however, leads to underestimate of the effect of multiple deflections (Fukushige & Makino 1994).

In the present paper, we investigate the luminosity change due to multiple lensing by integrating the orbits of photons (null-geodesics equations) in the model universe with uniformly distributed lensing objects. We found that the attenuation is significant. A source at $z = 4$ is typically attenuated by a factor of 2 under a reasonable assumption.

Our result can be understood by using the theory of exponential divergence of two light paths (Goodman, Heggie, & Hut 1993, hereafter GHH). GHH developed a theory to describe how the orbits of test particles with slightly different initial positions diverge from each other in a stellar system. According to their theory, the mechanism is summarized as follows. The scattering (lensing) angle χ of a photon by an encounter with a gravitational source of mass M is

$$\chi = \frac{4GM}{c^2 b}, \tag{1}$$

where c and b are the light velocity and the impact parameter of the encounter, respectively. When two photons are deflected by the encounter with a gravitational source, the change of the angle θ between the trajectories of the photons is obtained by differentiating equation (1):

$$\Delta\theta = \frac{4GM}{c^2 b^2}\Delta b, \tag{2}$$

where Δb is the separation of two photons before the encounter. Here, we assume that $\Delta b/b$ is small.

Now let us consider the case in which these two particles experience encounters with the impact parameter b and time interval $t_{\rm enc} = l/c$, where l is the mean free path. The evolution of the distance and the angle between two trajectories is given by the following finite difference equations

$$\begin{aligned} w_{k+1} &= w_k + l\theta_{k+1}, \\ \theta_{k+1} &= \theta_k + \frac{4GM}{c^2 b^2} w_k, \end{aligned} \tag{3}$$

where w_k and θ_k are the distance and angle between two particles just after kth encounter, respectively. The solution of equation (3) is given by

$$\begin{aligned} w_k &= w_1\left(1+\frac{4GMl}{c^2b^2}\right)^{k-1} \simeq w_1 \exp\left[\left(\frac{4GM}{t_{\rm enc}\, cb^2}\right)^{1/2} t\right], \\ \theta_k &= \theta_0 \frac{4GM}{c^2b^2}\left(1+\frac{4GMl}{c^2b^2}\right)^k \\ &\simeq \theta_0 \frac{1}{c}\left(\frac{4GM}{t_{\rm enc}\, cb^2}\right)^{1/2} \exp\left[\left(\frac{4GM}{t_{\rm enc}\, cb^2}\right)^{1/2} t\right]. \end{aligned} \tag{4}$$

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GHH found that the encounters with $b = w_{crit} \sim 5c^{-1/2}G^{1/2}m^{1/2}\rho^{-1/4}$ are most effective. Here, ρ is the mass density of the lensing objects. Fukushige et al. (1994a, b, hereafter FME) investigated how the light path is deflected by multiple lensing by numerical simulations. FME found that the e-folding time, τ, of the growth is given by $\tau \sim 0.22/(G\rho)^{1/2}$ for $w < w_{crit}$. When w becomes larger than this w_{crit}, the e-folding time becomes longer than the above value.

If we assume that the mass of a typical galaxy is $10^{12}\ M_\odot$ and that the density parameter, Ω_l, is ~ 0.1, the e-folding time τ of the exponential growth is comparable to the Hubble time. In an expanding universe, as we go back toward the epoch of the big bang, the timescale τ becomes shorter since ρ becomes higher. Photons emitted at high-redshift region, therefore, traveled a distance long enough to grow the separation between them. The critical angle w_{crit} is $\sim 0.3\ h^{-1/2}$ Mpc, which is much larger than the size of a galaxy. Therefore, this effect changes the shape and luminosity of high-redshift objects significantly.

In this *Letter*, we numerically calculate the image of objects in a model universe in which lensing objects are uniformly distributed. We found that the observed image of a light source is, in most cases, smaller than the true size of the source due to the exponential divergence of two light rays. This light source is attenuated, since the luminosity is proportional to the solid angle of its image as a consequence of the conservation of the intensity along the light path.

If we assume that the universe is flat ($\Omega_0 = 1$), the decrease of the luminosity for $z > 1$ is given by

$$\mu_{0.5} \propto z^{-1.3(\Omega_l)^{1/2}}, \tag{5}$$

where $\mu_{0.5}$ is the median value of attenuation ratio, μ and Ω_l is the density parameter of lensing objects, respectively. The attenuation ratio is defined as $\mu \equiv L_{obs}/L_{true}$, where L_{obs} and L_{true} are the observed luminosity and the true luminosity, respectively. Equation (5) is consistent with the growth factor of separation of two light paths obtained by FME. This result implies that the statistics of high-redshift objects is strongly biased by multiple gravitational lensing. For an example, the Hubble diagram could be shifted by ~ 1 mag at $z = 3$.

We describe the numerical method and models in § 2, our result in § 3, and implications on the statistics of high-redshift objects in § 4.

2. MODEL AND THE NUMERICAL METHOD

We approximated the gravitational field of a single source as the weak-gravity limit of the Schwarzschild solution. In this approximation, the gravity generated by multiple sources is represented by the superposition of the gravitational fields due to individual sources.

We calculated the image of a source by the following procedure. We first follow the orbit of a photon emitted from the observer and place the source at the end point of this orbit. Then we calculated the orbits of photons emitted from the observer in a rectangular grid. The image of source is constructed by picking up the photons that reached the source. Figure 1 illustrates our method.

We calculated trajectories of 10^3–10^4 photons to construct one image. The number of lensing objects N_g is 128. The mass of each lensing object is set to be equal; $m = M_{tot}/N_g$. The lensing objects are distributed randomly in the sphere of unit radius ($R = 1$). We used the system of units in which the gravitational constant G and the light velocity c are both unity. The

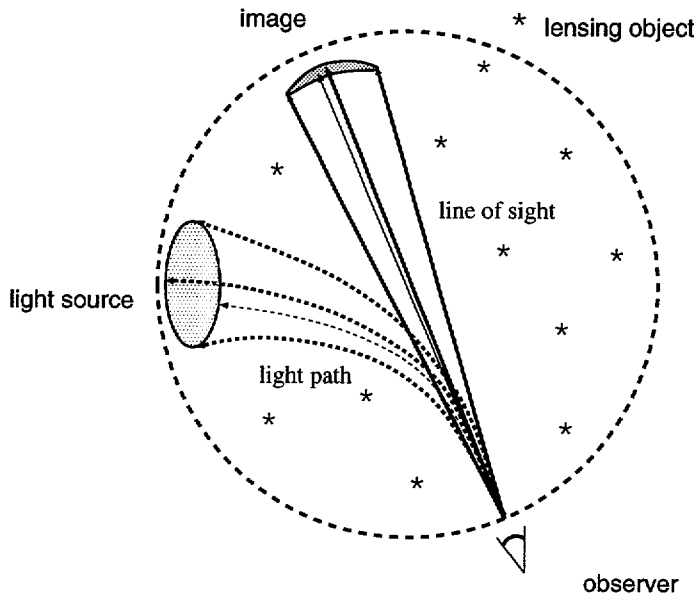

FIG. 1.—The image deformation due to lensing

observer is located at the position $(-1, 0, 0)$. The direction of the beam of photon is parallel to x-axis. The number of galaxies in the universe is much larger than 128. This difference, however, does not affect the results, since the timescale of growth is determined by the mass density ρ. It depends on neither the mass nor the number of the lensing objects. How results change when the distribution of the lensing objects is not uniform should be investigated. In this *Letter*, however, we do not discuss such cases, for simplicity.

The equation for the trajectory of a photon is given by

$$\frac{dx^0}{d\lambda} = \frac{1}{1+\phi},$$

$$\frac{d^2x^i}{d\lambda^2} = \frac{1}{1+\phi}\left[-\frac{1}{2}\left(\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\right)\frac{\partial\phi}{\partial x^i} + \left(\frac{dx^j}{d\lambda}\frac{\partial\phi}{\partial x^j}\right)\frac{dx^i}{d\lambda}\right],$$

$$(i, j = 1, 2, 3, \quad \mu = 0, 1, 2, 3, \quad \eta_{\mu\nu} = \delta_{\mu\nu}). \tag{6}$$

where λ is the affine parameter, ϕ is the potential, and $x^\mu (\mu = 0, 1, 2, 3)$ is (ct, x, y, z), respectively. The potential is

$$\phi = -\sum_{k=1}^{N_g}\frac{2m}{(|x - x_k|^2 + \epsilon^2)^{1/2}} + \frac{1}{2}\left(\frac{3M_{tot}}{R} - \frac{M_{tot}}{R^3}|x|^2\right), \tag{7}$$

where x is the position of a photon (x, y, z), m and x_k are the mass and position of the kth lensing object, N_g and M_{tot} are the number and total mass of lensing objects, and R is the radius of the sphere in which the lensing objects are distributed. The first term of equation (7) is the contribution of individual gravitational sources. Here, ϵ is a softening parameter which represents the size of the lensing objects. The second term is introduced to cancel the global harmonic potential of the sphere. This weak field approximation is appropriated for real lensing objects, such as galaxies and cluster of galaxies, since the depth of the potential well of such lensing objects is far smaller than c^2.

We obtained the trajectory $\{x^\mu\}$ of each photon by integrating equation (6). We used the fifth-order Runge-Kutta-

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Fehlberg formula with automatic step-size control. The number of steps is ~ 100 for one photon. The step size was determined so that the local error of energy conservation was smaller than 10^{-6}, which is small enough to calculate an image. The softening parameter is $\sim 10^{-3}$–10^{-2}, which is small enough not to affect on the result. Detailed description on our numerical method is given elsewhere.

We performed simulations for 11 different mass density ρ. The relation between the mass density ρ and the redshift z in an expanding universe at which the light source lies is given by

$$z = -1 + \exp\left[\sqrt{\frac{32\pi G\rho}{3\Omega_l}}\right], \quad (8)$$

for $\Omega_0 = 1$, where Ω_l is the density parameter of the mass in the lensing objects. For each ρ, we calculated 40 images of the source galaxy with different configurations of gravitational sources.

3. RESULTS

Figures 2*b*–2*e* show four set of pictures for an example of observed images of the circular light source with the radius 0.0015625 (Fig. 2*a*) for cases $\beta = 1.25$, 2.0, 3.6, and 5.2, where β is defined as $\beta \equiv (1+z)^{(\Omega_l)^{1/2}}$. We used an uniform disk as a light source (Fig. 2*a*). The images of galaxies are not so much distorted for $\beta > 1$, while they are strongly distorted for $\beta > 2$ ($z > 1$ for $\Omega_l = 1$).

In Figure 3, the cumulative fraction $S(\mu)$ of images is plotted against the attenuation ratio μ for each value of β. The attenuation ratio μ is calculated as $\mu = A_{\rm obs}/A_{\rm true}$, where $A_{\rm obs}$ is the solid angle of the perturbed image and $A_{\rm true}$ is that of the unperturbed image. The ratio of the observed luminosity to the true luminosity is equal to the ratio of solid angle of the perturbed image to that of the unperturbed image. Figure 3 shows that the images of distant galaxies are more strongly attenuated than those of nearby galaxies.

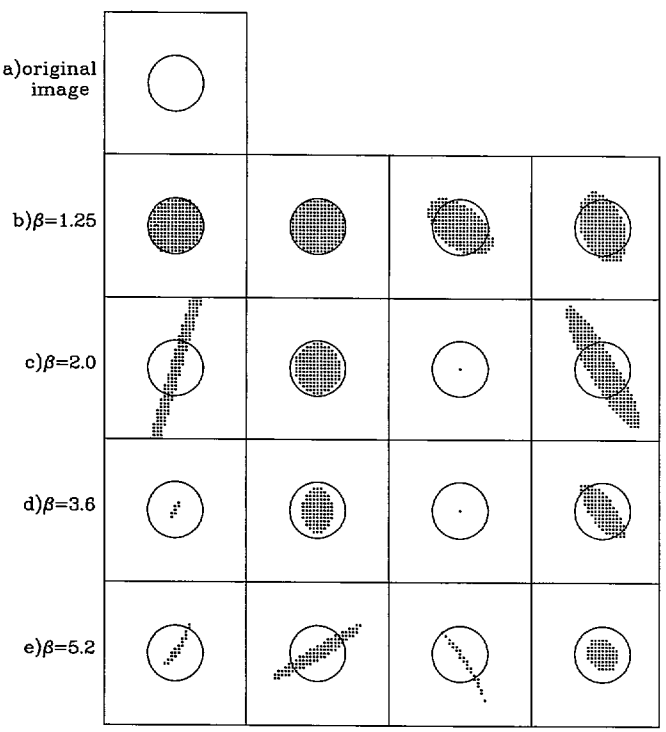

FIG. 2.—Observed images. (*a*) Solid circle shows the original image of the light source. Four examples of observed images for four cases of $\Omega_l = 0.2$ and (*b*) $\beta \equiv (1+z)^{(\Omega_l)^{1/2}} = 1.25$ (*c*) $\beta = 2.0$ (*d*) $\beta = 3.6$ (*e*) $\beta = 5.2$.

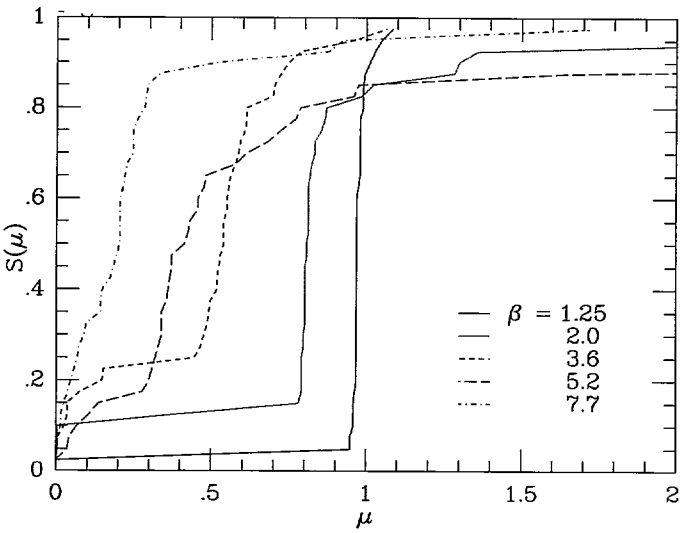

FIG. 3.—Cumulative fraction $S(\mu)$ of images as the function of the attenuation ratio μ for each case. Each curve corresponds to the case in which $\Omega_l = 0.2$, $\beta = 1.25$, 2.0, 3.6, 5.2, 7.7, respectively.

In Figure 4, the median value $\mu_{0.5}$ of the attenuation ratio is plotted against β. Here, $\mu_{0.5}$ is the value of μ for which $S(\mu) = 0.5$. Figure 4 shows that $\mu_{0.5} \sim (1+z)^{-1.3(\Omega_l)^{1/2}}$ for $z > 1$. For example, half of galaxies observed at $z = 6$ is attenuated by a factor of 2 for $\Omega_l = 0.2$.

There are a few strongly amplified sources in the high z cases, while most source objects in high z are attenuated. The existence of a few amplified galaxies is necessary to assure the conservation of the total luminosity. These amplified images might be observed as very luminous QSOs and BL Lac objects.

4. SUMMARY AND DISCUSSION

We have numerically investigated the effect of multiple gravitational lensing on the luminosity of distant galaxies and

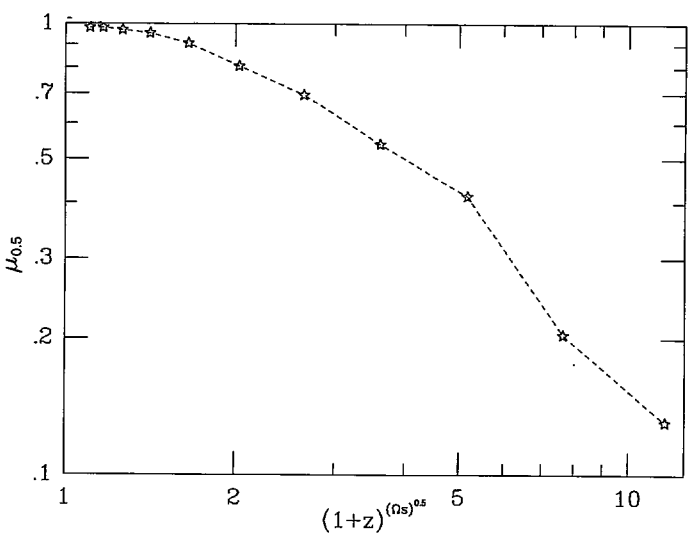

FIG. 4.—Relation between the redshift z of the light source and the median attenuation ratio $\mu_{0.5}$.

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QSOs. We obtained the distribution of the change of the luminosity as a function of the redshift. We found that the luminosity of the distant objects are significantly attenuated through multiple gravitational lensing. The attenuation ratio is larger for more distant sources.

The median value of apparent magnitude of distant galaxies decreases by

$$\Delta m(z) \simeq 3.25\sqrt{\Omega_l}\log(1+z)\,. \tag{10}$$

This attenuation of median value of apparent luminosities changes statistics of distant galaxies and QSOs. Our result implies that half of objects with $z \geq 3$ become fainter by 1 mag or more. Yoshii & Takahara (1988) showed that the difference between luminosity of giant ellipticals at $z = 2$ for models with $\Omega_0 = 1.0$ and $\Omega_0 = 0.04$ is $1 \sim 2$ magnitude. Therefore, the effect of the gravitational lensing is comparable to that of the variation in Ω_0. Other statistics, such as the distribution of QSOs and the number count of galaxies, would also be strongly affected.

We thank D. Sugimoto and T. Fukushige for helpful discussion. We also thank P. Hut for stimulating discussions. This research is partially supported by Grant-in-Aid for Specially Promoted Research of the Ministry of Education, Sciences and Culture.

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