Post-Collapse evolution of globular clusters

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ABSTRACT

A number of globular clusters appear to have undergone core collapse, in the sense that their predicted collapse time is much shorter than their current age. Simulations using gas models and Fokker-Planck approximation have shown that the central density of a globular cluster after the collapse undergoes nonlinear oscillation with large amplitude (gravothermal oscillation). However, whether such an oscillation actually takes place in a real N-body system has remained unsolved, because an N-body simulation with a sufficiently high resolution would have required the computing resource of the order of several Gflops·years. In the present paper, we report the result of such a simulation, performed on a dedicated special-purpose computer GRAPE-4. We simulated the evolution of isolated point-mass systems with up to 32,768 particles. The largest number of particles reported previously is 10,000. We confirmed that gravothermal oscillation takes place in an N-body system. The expansion phase shows all signatures that are considered as the evidences of the gravothermal nature of the oscillation. At the maximum expansion, the core radius is $\sim 1\%$ of the half-mass radius for the run with 32,768 particles. The maximum core size r_c depends on N, as $< r_c > \propto N^{-1/3}$.

1. Introduction

A number of globular clusters appear to have undergone core collapse (Cohn and Hut 1984, Hut and Djogovski 1992). Djorgovski and King (1986) showed that 15% of galactic globular clusters have unresolved density cusps. Recent observations with HST have demonstrated that some of these clusters have the core sizes smaller than 0.03 arcsec, which is as far as one can go with present technique (*e.g.*, Sosin and King 1995, Yanny *et al.* 1994).

The theoretical study of the evolution after the core collapse was pioneered by Henon (1975), who incorporated the energy production by binaries into his Monte-Carlo calculation. He found that the whole cluster expanded homologously in the thermal timescale. Similar results were obtained by gaseous models and Fokker-Planck calculations.

Sugimoto and Bettwieser (1983, Bettwieser and Sugimoto 1984) found that the post-collapse expansion is unstable if the energy production is inefficient. In the unstable case, the central density showed an oscillation with a large amplitude. They called this oscillation gravothermal oscillation. They modelled the post-collapse evolution of globular clusters using a conducting gas sphere with artificial energy production. The efficiency of the energy production is related to the total number of particles N, and for large N efficiency is small. Thus, their result implies an N-body system with number of particles larger than a critical number should show the gravothermal oscillation.

Since several other calculations based on similar models did not reproduce the result of Sugimoto and Bettwieser (1983), the validity of their result had been controversial for a few years after their first paper. However, calculations with improved accuracy confirmed that the oscillation takes place in both the gaseous models and Fokker-Planck (FP) calculations(Goodman 1987, Heggie and Ramamani 1989, Cohn *et al.* 1989).

What have been found in gas models and FP calculations are summarized as follows. There is a homologously expanding solution for the post-collapse evolution of the cluster (Goodman 1984). In this solution, the size of the core is proportional to $N^{-1/3}$. Thus, the core is smaller for larger number of particles. If the core is too small, this homologous expansion becomes unstable in a way similar to the way in which an isothermal sphere is unstable, and the core starts to show an oscillating behavior. The critical number of particles is around 7,000 (Goodman 1987). If the total number of particle is slightly larger than this critical value, the oscillation is regular. However, at $N \simeq 10,000$, "period doubling" takes place and the density shows oscillation with double peaks. For $N > 50,000$, oscillation becomes apparently chaotic (Heggie and Ramamani 1989, Breeden *et al.* 1994).

Both the contraction and the expansion are driven by the gravothermal instability, and the energy production by binaries acts as the trigger to start the expansion. During the expansion, the thermal energy flows in from the halo to the core, and this heat flux is supported by so-called "temperature inversion". This temperature inversion itself is formed through the expansion of the core driven by the binary heating in the following way.

Binaries deposit the energy to the core through "indirect heating" (Hut 1985). When a binary hardens through interaction with a third star, that star is likely to be kicked out of the core, because the recoil velocity is much larger than the velocity dispersion of the core. As a result, the binding energy of the core becomes smaller. The core expands to reestablish the dynamical equilibrium. This expansion causes the velocity dispersion of the core to decrease. Thus the central temperature becomes lower than the temperature of the region just outside the core.

Once the central temperature becomes lower than the temperature around the core, the heat starts to flow inward. This inward heat flux let the core expand further, and the temperature of the core continues to decrease. This self-supported expansion can continue as far as the temperature gradient outside the core is small. In practice, however, temperature gradient becomes larger as the core size becomes larger, and the temperature inversion vanishes at a certain core size. After that, the core starts to recollapse.

The critical number of particles for multi-component systems is larger than that for single-component systems, and for a "realistic" mass function, the critical number of particle can be as large as 3×10^5 (Murphy *et al.* 1990).

The question whether such gravothermal oscillation would takes place in real globular clusters or in N-body systems has not been settled yet. Both gaseous models and FP models have many simplifying assumptions that might make the evolution completely different from that of a real cluster. For example, both assume spherical symmetry, while in N-body simulations cores are known to wander around (Makino and Sugimoto 1987, Heggie *et al.* 1994). Both assume that the energy production by binaries is smooth, while in N-body system binaries are formed stochastically and emit energy intermittently.

An N-body simulation with a sufficiently large number of particles has been impossible, simply because the requirement for computer power has been excessive. Makino *et al.* (1986) performed 100-body simulations of an equal-mass cluster, using both softened and unsoftened potentials. They used a softened potential in order to reduce the energy production from binaries. They found oscillatory behavior for both cases, but with larger amplitude for softened potential. They argued that the fact that the amplitude of oscillation is larger for reduced energy production implies that the observed oscillation is driven by the gravothermal instability and not purely by the heating by binary. McMillan and Lightman (1984, McMillan 1986a) developed hybrid calculation code in which the central region of a cluster is treated as an N-body system while the outer region is treated in FP formalism. They observed oscillatory behavior but concluded that it was not gravothermal. Inagaki (1986) performed 1000- and 3000-body simulations but found no sign of oscillation.

He used standard Salpeter mass function as the distribution of the masses of particles. Makino (1989) performed simulation of a 3000-body equal-mass system and saw some signs of oscillation, including the temperature inversion. Spurzem and Aarseth (1996) performed 10,000 body simulation and saw oscillatory behavior, but they did not see any clear evidence of the gravothermal oscillation. N-body simulations in the last decade can be summarized as follows: The central density showed some oscillatory behavior, and its amplitude is seemingly larger for larger number of particles. However, each expansion phase is associated with large energy input from one or a few binaries. As a result, whether the expansion is driven by instability or by energy input has been unclear. In addition, in all but one simulations, the number of particles was smaller than the critical value of 7,000. The 10,000 body simulation by Spurzem and Aarseth (1996) was not long enough to draw clear conclusion. Therefore it was not clear whether oscillatory behavior is caused by gravothermal instability or not.

Heggie (1989, see also Heggie *et al.* 1994) tried to see whether gravothermal expansion would take place if one constructs an N-body system that has an initial temperature inversion. They constructed an N-body system which mimics the density and temperature structures of the expansion phase of the gas model calculation by Heggie and Ramamani (1989), and follow the evolution of the N-body system. They found that the core expands in the thermal timescale. Thus, they at least proved that the core of an N-body system can expand gravothermally.

In the present paper, we describe the result of N-body simulations with number of particles 2,048–32,768. This is the first calculation with the number of particles well beyond the critical number of 7,000 which covers sufficiently long time after the collapse to determine the nature of the post-collapse evolution. All simulations were performed on GRAPE-4 (Taiji *et al.* 1996), a special-purpose computer for collisional N-body simulations. Our main results are the following. First, for large N, the core density and core mass exhibited oscillations with large amplitude. The core mass at the maximum expansion is in good agreement with FP or gas model results (1% of the total mass for 16k- and 32k-particle runs). Second, we confirmed that the observed oscillation is driven by gravothermal instability. Several long expanding periods without any energy input were observed. The temperature inversion was visible in such expansion phases. The behavior of the core density is strikingly similar to the result of FP calculations with stochastic heat source (Takahashi and Inagaki 1991), which strongly suggests that the mechanism is the same. The trajectory in the central density-central temperature plane clearly indicates that the inward heat flux supports the expansion.

The structure of this paper is the following. In section 2, we describe the initial model, numerical method and the computer used. In section 3 we present the result. Section 4 is for discussion.

2. Model and Numerical method

2.1. Initial models and the system of units

We followed the evolution of isolated systems of point-mass particles. For all calculations, we used random realizations of the Plummer model as the initial condition. The system of units we adopted is the standard (Heggie) units (Heggie and Mathieu 1986), in which $G = 1$, $M = 1$, and $E = -1/4$, where G is the gravitational constant, M is the total mass of the cluster, and E is the total energy of the cluster. All particles have the same mass $m = 1/N$, where N is the total number of particles. In this unit, the half-mass crossing time t_{hc} is $2\sqrt{2}$.

2.2. Numerical method

For all calculations, we used NBODY4 (Aarseth 1985, 1996), modified for GRAPE-4 (Taiji *et al.* 1996). The numerical integration scheme adopted in NBODY4 is the 4th order Hermite scheme (Makino 1991a, Makino and Aarseth 1992) with the individual (hierarchical) timestep algorithm (McMillan 1986b, Makino 1991b) to use the GRAPE hardware or parallel/vector computers efficiently. Close two-body encounters and stable binaries are handled by KS regularization (Kustaanheimo and Stiefel 1965). Special treatment for compact few-body subsystems is also possible.

The 4th-order Hermite scheme has several advantages over traditional ABM type schemes. It is a self-starting scheme and therefore much easier to implement than the multistep ABM scheme of the same order. Its local truncation error has much smaller coefficient, which allows a larger timestep for the same accuracy. On general-purpose computers, this advantage is partly cancelled by the additional CPU time to calculate the first time derivative, but on GRAPE-type machines this is an significant advantage (Makino and Aarseth 1992, Makino *et al.* 1993).

The output are taken at intervals of a fixed time, which is a certain fraction of the crossing time. We recorded central density, core radius, number of particles in the core, radii of Lagrangian shells, velocity dispersion within Lagrangian shells, binaries and their binding energies. The core parameters are calculated following Casertano and Hut (1985), with modifications described in McMillan *et al.* (1990).

The accuracy parameter of the time integration is adjusted so that the energy error between two outputs is smaller than a certain prescribed value. The value we used is $1 \times 10^{-5} \sim 1 \times 10^{-6}$ depending on N. We required higher accuracy for larger N, since the duration of the simulation is longer. When the energy error is very large, the program automatically reads the output at the previous checkpoint and restarts with a reduced accuracy parameter.

The N-body simulation of point-mass systems with a large number of particles poses several technical problems beside the necessary computing power. The difficulty comes from the wide range of the timescale. The critical separation of two particles at which we apply the KS regularization is about $1/N$. The timestep for particles involved is of the order of $1/(100N^{1.5})$. On the other hand, the timescale of the evolution of the system is of the order of N. Thus, the timesteps of particles that are integrated in KS formalism is $1/100N^{2.5}$ of the system time, which, for $N \sim 10^5$, about as small as the limit of double precision numbers. Note that this estimate is valid for a soft binary. A hard binary would require timesteps smaller by several orders of magnitude than described above. Of course, a clever treatment can overcome this kind of difficulties, but the amount of programming work and the complexity of the resulting code would be considerable. The simplest solution would be to use the longer number format (*e.g.*, the quadruple precision). This would have been a viable solution in '70s, when mainframe machines could handle quadruple precision arithmetic reasonably fast. Unfortunately, most of present RISC computers are extremely slow at quadruple precision arithmetic.

2.3. Hardware and calculation cost

For all calculations, we used GRAPE-4 (Taiji *et al.* 1996). The GRAPE-4 is a special-purpose computer designed to accelerate the N-body simulation using the Hermite integrator and hierarchical timestep algorithm. The total system consists of 1692 pipeline processor chips and has the theoretical peak speed of 1.08 Tflops. The simulations reported in the present paper were performed while the assemblage and testing of the GRAPE-4 system were underway. Thus the number of processors varies during the calculation. For most of the 32k particle run, we used one quarter of the system with 423 processor chips, which has the peak speed of 270 Gflops. Calculations with smaller N were performed while the available number of processors was smaller.

The actual performance depends on many factors, but most strongly on the number of particles. With the present host computer (DEC 3000/900), average speed we got for 32k-particle run was about 50 Gflops. This 32k-particle run took about three CPU months. In comparison, the 10k-particle run reported by Spurzem and Aarseth (1996) took two CPU months on a Cray YMP. Since the calculation cost of globular cluster simulation is proportional to $N^{3.3}$, one quarter of GRAPE-4 is effectively 50 times faster than a Cray YMP.

At present, a 32k-particle system is about the largest system we can try for equal-mass case. Since the calculation speed of GRAPE-4 is still limited by the speed of the host computer, the actual calculation speed is roughly proportional to N for $N < 2 \times 10^5$. Thus, the CPU time for globular cluster simulation on GRAPE-4 is proportional to $N^{2.3}$. A 64k-particle run would take about one year.

The above estimate is for an equal-mass isolated cluster of point-mass particles. The computational cost of more realistic calculations depends on many factors. Among them, the mass spectrum and the presence of primordial binaries are most important, and have adverse effects.

Systems with a mass spectrum evolves faster than equal-mass systems (see, *e.g.*, Inagaki and Wiyanto 1984, Murphy and Cohn 1988). Therefore it is possible to handle a larger system. For example, if the evolution of the system is p times faster, GRAPE-4 can handle the number of particles $\sqrt{p}$ times larger. Thus, if the collapse is 4 times faster, GRAPE-4 can finish a 32k-particle run in 3 weeks or 64k-particle run in 3 months.

The presence of the primordial binaries would increase the CPU time. At present, however, actual cost is difficult to estimate because the calculation cost depends strongly on the core size, of which we have rather little knowledge (see section 4). It should be noted that the number of floating point operations used to follow the life of a binary depends only weakly on the total number of particles in the system (Makino and Hut 1990). Therefore, while the CPU time for the orbit integration of single stars would increase as $N^{2.3}$, the cost of binaries would increases only as N. It is likely that for large N the cost of binaries is relatively small, even for simulations with primordial binaries.

3. Result

3.1. The core oscillation

Figure 1 shows the time evolution of the central density for all runs. The time is scaled so that thermal timescale is the same for all runs. The scaling factor is $t_r(1000)/t_r(N) = 212.75 \log(0.11N)/N$ (Giersz and Heggie 1994). The core density shows oscillations with a large amplitude in calculations with large N (>16k). No matter what is the real nature of this oscillation, it is at least clear that the core density of the N-body system shows oscillation with the amplitude comparable to that observed in gas models or FP calculations.

For runs with small N (2k and 4k), there are some oscillation-like features but they are hardly distinguishable from fluctuations. This result is similar to that of previous studies (*e.g.* Makino 1989). For large values of N, however, the oscillation with large amplitude is clearly visible.

Note that in figure 1 there is no clear transition from stable expansion to oscillation or from regular oscillation to chaotic oscillation, which were observed in gas and FP models (Heggie and Ramamani 1989, Breeden *et al.* 1994). The reason is that binaries emit energy intermittently (Makino and Sugimoto 1987). Takahashi and Inagaki (1991) incorporated this stochastic nature of the energy source to their FP model and found that the core density shows chaotic oscillatory behavior even if the energy production rate is larger than the critical value for transition between stable expansion and regular oscillation. They also found that the amplitude of the oscillation is smaller for larger energy input (smaller N), which is consistent with the present result. In fact, it would be difficult to distinguish between our N-body calculation results from their stochastic FP results. Only visible difference is that their result is smoother while the central density is low. This is because the FP result is perfectly smooth as far as there is no binary heating.

3.2. Analysis of the 32k run

In the previous section, we overview the post-collapse evolution of N-body point-mass systems with 2k-32k particles. In this section, we take a closer look of the 32k-particle simulation to see whether we can find direct signature of the gravothermal expansion. It is generally believed that a long expansion phase without significant energy input and a temperature inversion during the long expansion phase are the most direct signatures of the gravothermal expansion (Bettwieser and Sugimoto 1984, McMillan and Engle 1996). In this section we investigate both of them.

Figure 2 shows an enlarged view of the time variation of the core radius as compared with the sum of binding energies of all binaries. It is clear that most of the energy is generated when the core is very small.

The continued expansion without energy input from binaries has been considered to be the most direct signature of the gravothermal oscillation. In figure 2, we clearly see three such expansions, for $t = 6700 - 6740$, $t = 6860 - 6920$, and $t = 7220 - 7280$. All these expansions continue for more than 10 half-mass crossing times, which is hundreds of the core relaxation time. These expansions cannot be driven simply by binary heating. If the expansion were driven only by binary heating, it could not continue without energy input for the timescale more than 100 times longer than the core relaxation time.

Figure 3 shows temperature profiles for the contracting and expanding phases. Near the end of the expanding phase the temperature inversion of the order of 5% is clearly visible. Since these profiles are time-averaged over 10 time units (80 snapshots), the actual inversion might be somewhat stronger. Note that the temperature inversion is visible only near the end of long expansion phases also in gas model and FP calculations (Bettwieser and Sugimoto 1984, Cohn *et al.* 1989).

Figure 4 shows the relation between the central density and the central velocity dispersion. The trajectory shows striking resemblance to what obtained by gas model calculation (Goodman 1987). The fact that the trajectory shows clockwise rotations means that this is a refrigeration cycle, in which the central region absorbs heat when the temperature is low, and release heat when the temperature is high (Bettwieser and Sugimoto 1984, Bettwieser 1985). In particular, the later phase of the large expansions (indicated by the arrow marked "B") is nearly isothermal. Since the binding energy of binaries is unchanged during this phase, this nearly isothermal expansion is driven by the heat supplied from outside the core. In other words, the expansion is gravothermal.

3.3. The core size — result and interpretation

Figure 5 shows the evolution of the number of particles in the core. For all runs, the number of particles at the maximum contraction is of the order of 10, while that at the maximum expansion is $\sim 1\%$ of the total number of particles. This result is again in good agreement with gas models and FP calculations (*e.g.*, Heggie and Ramamani 1989 Breeden *et al.* 1994).

Figure 6 shows the fraction of time for which the number of particles in the core is smaller than the value N_c as a function of N_c/N, for the post-collapse phase. In other words, this figure shows the cumulative distribution of the core mass. For $N > 8192$, the median core mass is around 0.5-0.6 %. This corresponds to $r_c/r_h \sim 0.01$. For $N = 32,768$, the core mass is somewhat smaller than that for 16k or 8k runs.

Whether the core size in the gravothermal oscillation phase depends on the total number of particles or not is not well understood. Bettwieser and Sugimoto (1984) argued that clusters spend most of the time in the most expanded state, and that the structure of the cluster at the maximum expansion does not depend on the total number of particles. However, it is clear from figure 6 that the cumulative time is roughly proportional to the core radius, and that the coefficient is larger for larger N. In other words, the size of the core depends on N.

Simple theoretical argument suggests that the core size must depend on N. The time-averaged energy production in the core would be the same as that for the stable expansion (Goodman 1987, Heggie and Ramamani 1989). This requirement pose a constraint on the time-averaged core size. The energy production rate is expressed as

$$\frac{dE}{dt} \propto M_c \rho_c^2, \tag{1}$$

where M_c and ρ_c are the core mass and core density. Here we ignored the dependence on the velocity dispersion, since the inner part is almost isothermal. For M_c and and the core radius, r_c, we have the relation $M_c \propto r_c$, again because the inner part of the cluster is almost isothermal. Thus we have

$$\frac{dE}{dt} \propto M_c^{-3}. \tag{2}$$

The time-averaged energy production rate must be equal to that of the homologous expansion solution obtained by (Goodman 1984). The core size for the homologous expansion is $N^{-2/3}$ (Goodman 1984, Giersz and Heggie 1993). Thus we obtain

$$< M_c^{-3} >^{-1/3} \propto N^{-2/3}, \tag{3}$$

where $< x >$ means the time averaging.

Figure 7 shows the arithmetic mean and the maximum of the core mass after the collapse as functions of N. We also plot the quantity $M_e =< M_c^{-3} >^{-1/3}$, since we have the theoretical prediction only for this quantity. For maximum core mass, the FP result by Breeden *et al.* (1994) are plotted for $N = 5 \times 10^4$ and 10^6. These values are read by eye from their figure 5. The quantity M_e indeed shows reasonable agreement with the theoretical prediction of equation (3).

However, the average and maximum core radii shows the dependence noticeably weaker than the theoretical prediction. It should be noted that our N-body results for the maximum core mass and the FP results of Breeden *et al.* (1994) are on one line expressed as $M_c \propto N^{-1/3}$. From figure 7, the core mass at the maximum expansion is approximated as

$$M_{c,\max} \sim 0.006 \left(\frac{N}{10^5}\right)^{-1/3} M. \tag{4}$$

The slope of the average core size seems to be somewhat steeper.

Qualitatively, it is natural that the ratio between the maximum core mass and M_e is larger for larger N, since the ratio between the minimum core mass and M_e is larger for larger N. The minimum core mass is $O(1/N)$, since the number of particles in the core at the core bounce is always around 10-30. In other words, the energy production rate at the core bounce is larger than the time-averaged rate by a factor proportional to N. Thus, roughly speaking, the core mass has to stay at the value larger than the value at the core bounce for most of time. The typical relaxation timescale of the core not at the maximum contraction is, therefore, N times longer than that at the maximum contraction. Since the relaxation time is proportional to M_c^2, this implies that the typical core mass is $N^{-1/2}$. This is in fact a reasonable fit for the average core mass in figure 7.

One interesting question is whether it is possible to distinguish the core in the gravothermal oscillation and the core dominated by primordial binaries. The theoretical prediction by Goodman and Hut (1989) gives $r_c/r_h \sim 0.02$. The Fokker-Planck calculation by Gao *et al.* (1991) gave similar result. On the other hand, N-body simulations by McMillan *et al.* (1990) gave $r_c/r_h \sim 0.2$. McMillan *et al.* (1990) used 1136 particles. If their N-body results can be extrapolated to larger values of N, cores with primordial binaries and cores in gravothermal oscillation would be clearly distinguishable. On the other hand, FP result by Gao *et al.* (1991) indicates that the typical core size at the "primordial binary burning" phase and that at the gravothermal oscillation phase would not be much different. To obtain a definitive answer, we need to perform simulations of clusters with primordial binaries using the number of particles larger than that employed by McMillan *et al.* (1990).

3.4. The core size — comparison with observations

Djorgovski and King (1986) showed that 15% of galactic globular clusters have unresolved density cusp. They classified these clusters as "post core collapse" (PCC). Recent ground-based observations (Lugger *et al.* 1995) and HST results (Sosin and King 1995, Guhathakurta *et al.* 1994, Yanny *et al.* 1994) demonstrated that some of the PCC clusters have the cusps unresolved even with HST (NGC6624, M15). For these clusters, the surface density profile shows the power law down to 0.3 arcsec. Other clusters (M30), though the slope seems to levels off toward the center, turned out to be difficult to determine the slope accurately because there are too few stars. In the case of M30, the core radius can be anywhere smaller than 1.5 arcsec.

Many researchers claimed that there is a good agreement between the theory and observation of the core radius. For example, Djorgovski and Meylan (1994) found that no observed cluster has the core radius smaller than 1% of the half-light radius, using the data compiled by Trager *et al.* (1993), and argued this to be in good agreement with the model that assumes the binary-dominated core for PCC clusters (Vesperini and Chernoff 1994).

This apparently good agreement between the "observation" and "theory" is actually due to the fact that both of them overestimated the core size. On the observational side, High-resolution observations have shown that the real core size of the PCC clusters is significantly smaller than what is assumed in Djorgovski and Meylan (1994). Trager *et al.* (1993) adopted the HWHM as the core size of PCC clusters. Since the HWHM has no relation to the real core radius of the PCC clusters, it is quite natural that observations with higher resolution obtained smaller core radii. On the theoretical side, our present result suggest that the typical size of the core in the gravothermal oscillation phase is significantly smaller than the previous claims.

Our result suggests that the typical size of the core in the gravothermal oscillation phase is smaller than previous estimates because of the following two reasons. First, the time-averaged core size is about a half of the maximum size. Second, the core size depends on N as $N^{-1/\alpha}$, with index alpha $2 \sim 3$.

For NGC6624 and M15, core sizes seem to be too small for a stably expanding cluster, no matter what is the heat source. So it is highly likely that they are undergoing the gravothermal oscillation.

4. Discussion

We performed direct N-body simulation of the post-collapse evolution of globular clusters. We confirmed that the gravothermal oscillation actually takes place in a point-mass N-body system.

Whether real globular clusters undergo gravothermal oscillation or not is a question which requires further research. If clusters contain many primordial binaries, even after the core collapse they might still be burning the primordial binaries. In addition, the effect of two-body binaries to the evolution of the cluster is still unclear. In fact, the cross section for the binary formation by tidal capture is not fully understood yet (Mardling 1995a, 1995b).

The most straightforward way to study the effect of primordial binaries or two-body capture is the direct N-body simulation. In principle, we can put primordial binaries and their evolution into FP calculation as Gao *et al.* (1991) did. However, the standard one-dimensional FP calculation, which assumes the isotropic distribution, is not appropriate to follow the binary population, since most binaries are formed in the core and their orbits are nearly radial. Thus, we have to solve the FP equation at least in three dimensions (E, J and the binary binding energy E_b). It would require prohibitively large computer power. In addition, reliable two-dimensional FP calculation has become possible only very recently (Takahashi 1995). One could also use the Monte-Carlo approach, but its result must be compared with N-body simulation anyway.

We demonstrated that the N-body simulation with the number of particles close to real globular clusters is now possible, thanks to the extremely powerful special-purpose computer and its full-time availability. We are now able to use the direct N-body simulation to study various aspects of evolution of globular clusters.

Our 32k-particle calculation took about three months of CPU time on 1/4 of GRAPE-4. If we tried to do a similar calculation on a Cray T90 vector supercomputer, it would have taken several years of CPU time. It is simply impossible to do on present-day supercomputers. If we want to finish the calculation in, say, one CPU month, we need a computer 50-100 times faster than a Cray T90, which will be available in 10 years from now.

The CPU time of three months is still very long. However, for many simulations, we do not need as many as 32k particles. A 16k-particle calculation was finished in 2-3 weeks, on 1/8 of GRAPE-4. Thus to run many simulations of 16k particle system is now practical.

If we can continue the development of the special-purpose computer, we will have a system 100-1000 times faster than present GRAPE-4 in the next five years. Such a system will make it possible to run 50-100k particle simulation routinely, while 500k-1M particle simulation will still take months.

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Fig. 1.— The logarithm of the central density plotted as a function of the scaled N-body time. Curves for different values of N are vertically shifted by 3 units.

Fig. 2.— The core radius (top) and the binding energy of binaries (bottom) as a function of time for the 32k-particle run.

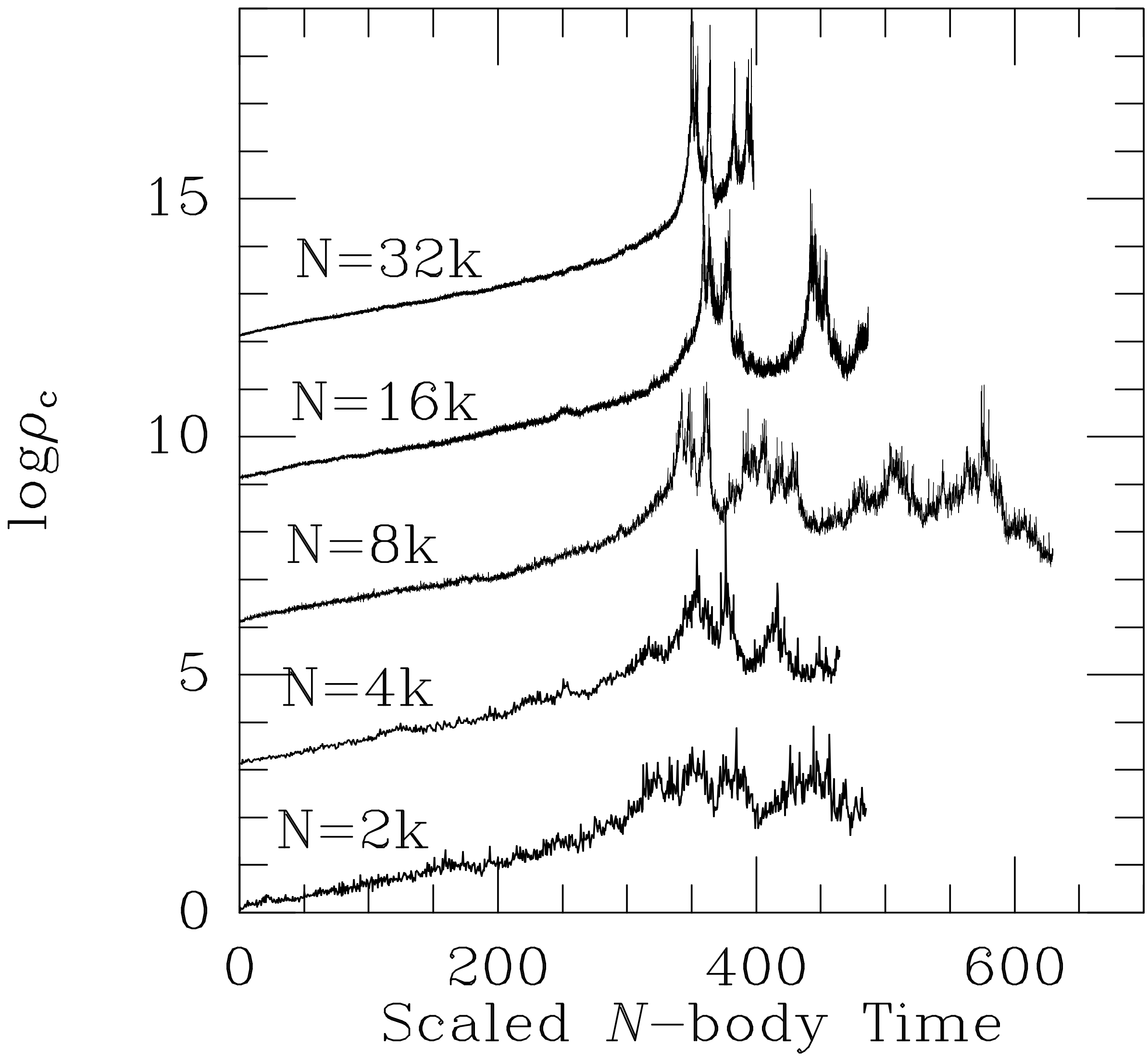
Fig. 3.— Velocity dispersion profiles for (a) contracting and (b) expanding phases. Each profile is obtained by time averaging over 80 snapshots (10 time units). The time interval between curves is 5 time units.

Fig. 4.— The change of the central density and the central velocity dispersion. Each data points is time-averaged value over 80 snapshots. Arrows indicate the direction of evolution.

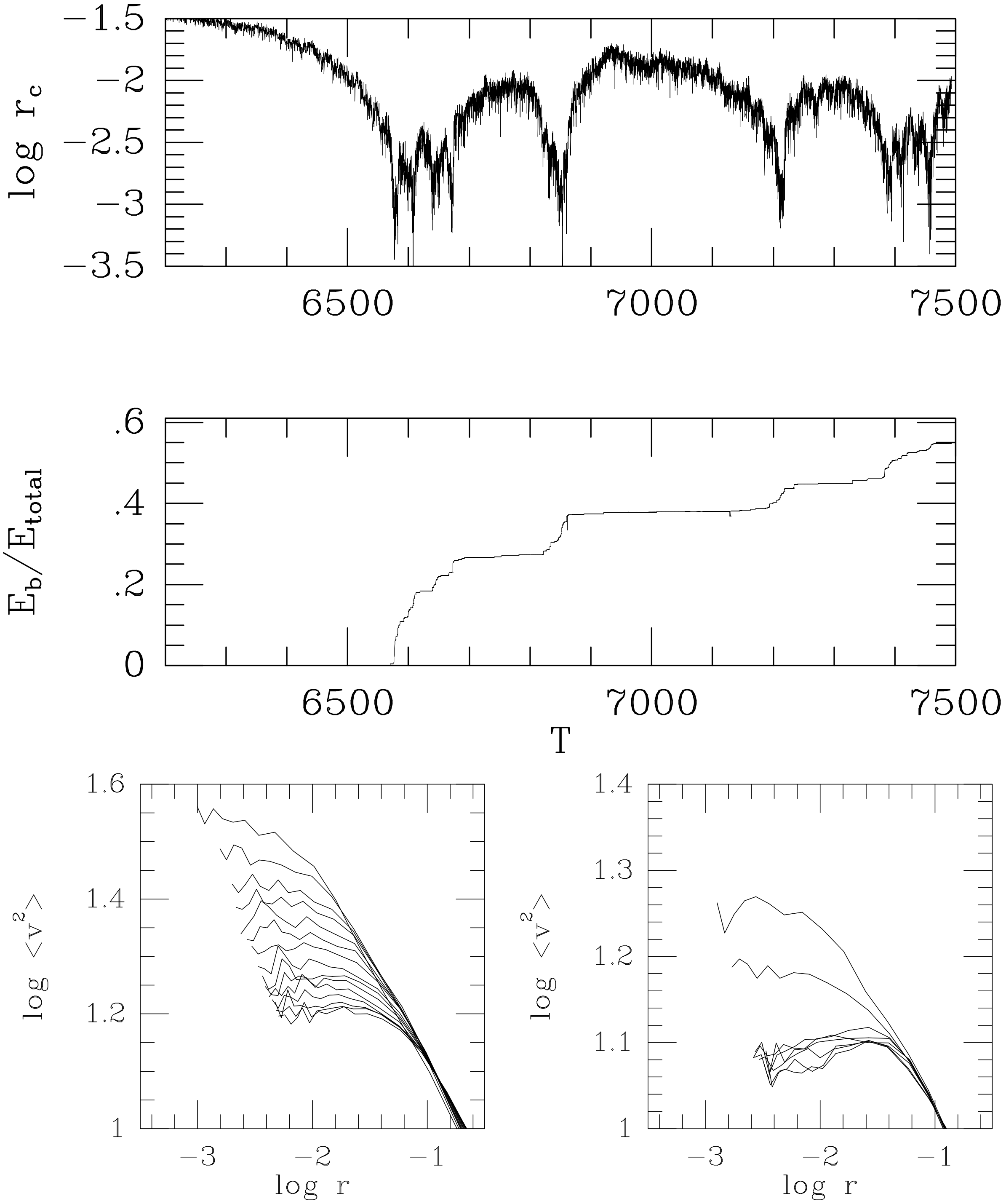
Fig. 5.— The number of particles in the core as the function of the scaled time, for simulations with 2k, 4k, 16k, and 32k particles

Fig. 6.— Fraction of time for which the number of particles in the core is smaller than N_c, as a function of N_c/N.

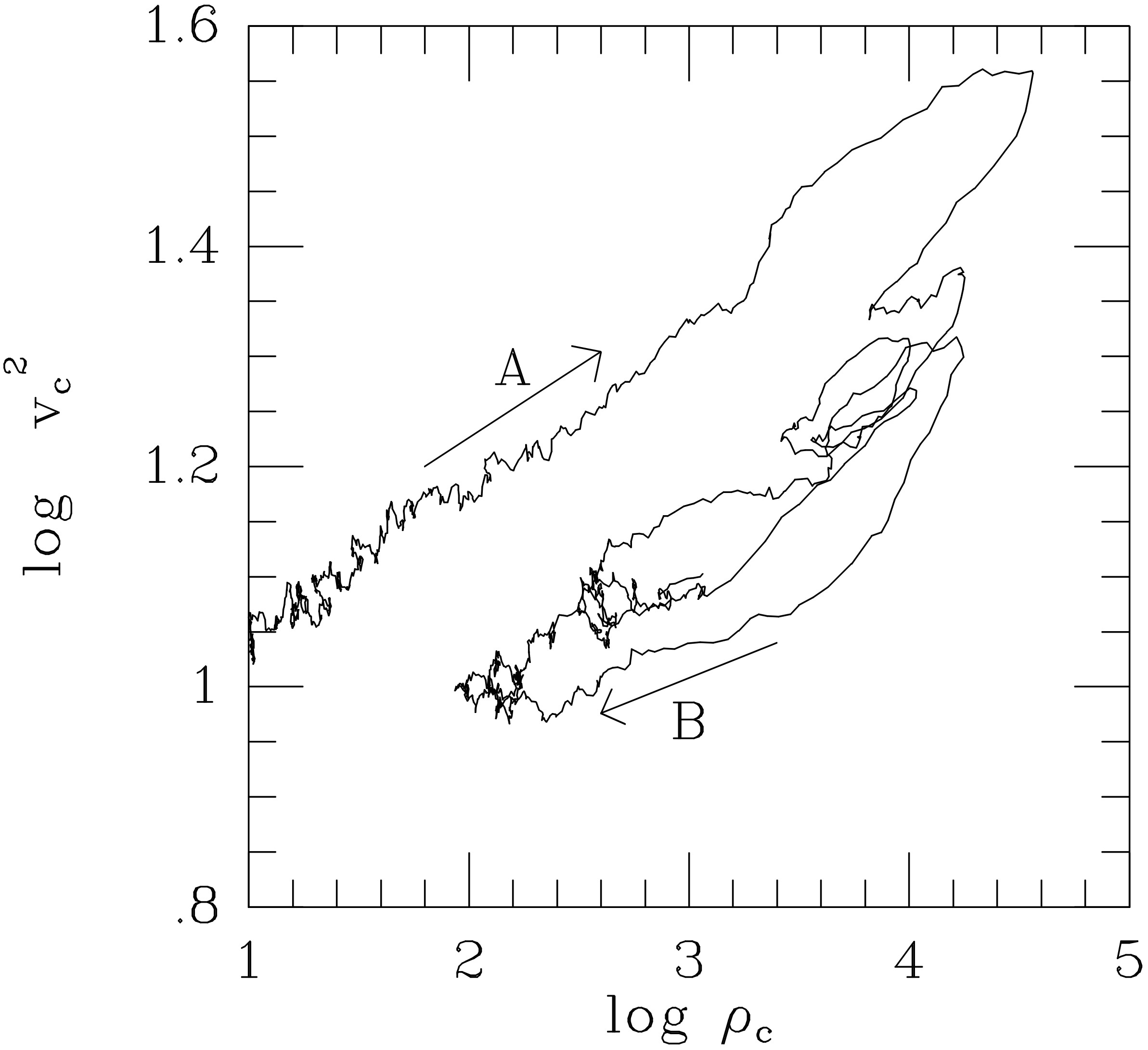
Fig. 7.— Core mass plotted as a function of the total number of particles N. Filled squares are M_e (see text). Filled crosses are arithmetic means. Filled circles are maximum values from present N-body simulations. Open circles are maximum values from FP calculations of Breeden *et al.* (1994). Straight lines indicates $M_c \propto N^{-1/3}$ (top) and $M_c \propto N^{-1/3}$ (bottom), respectively.

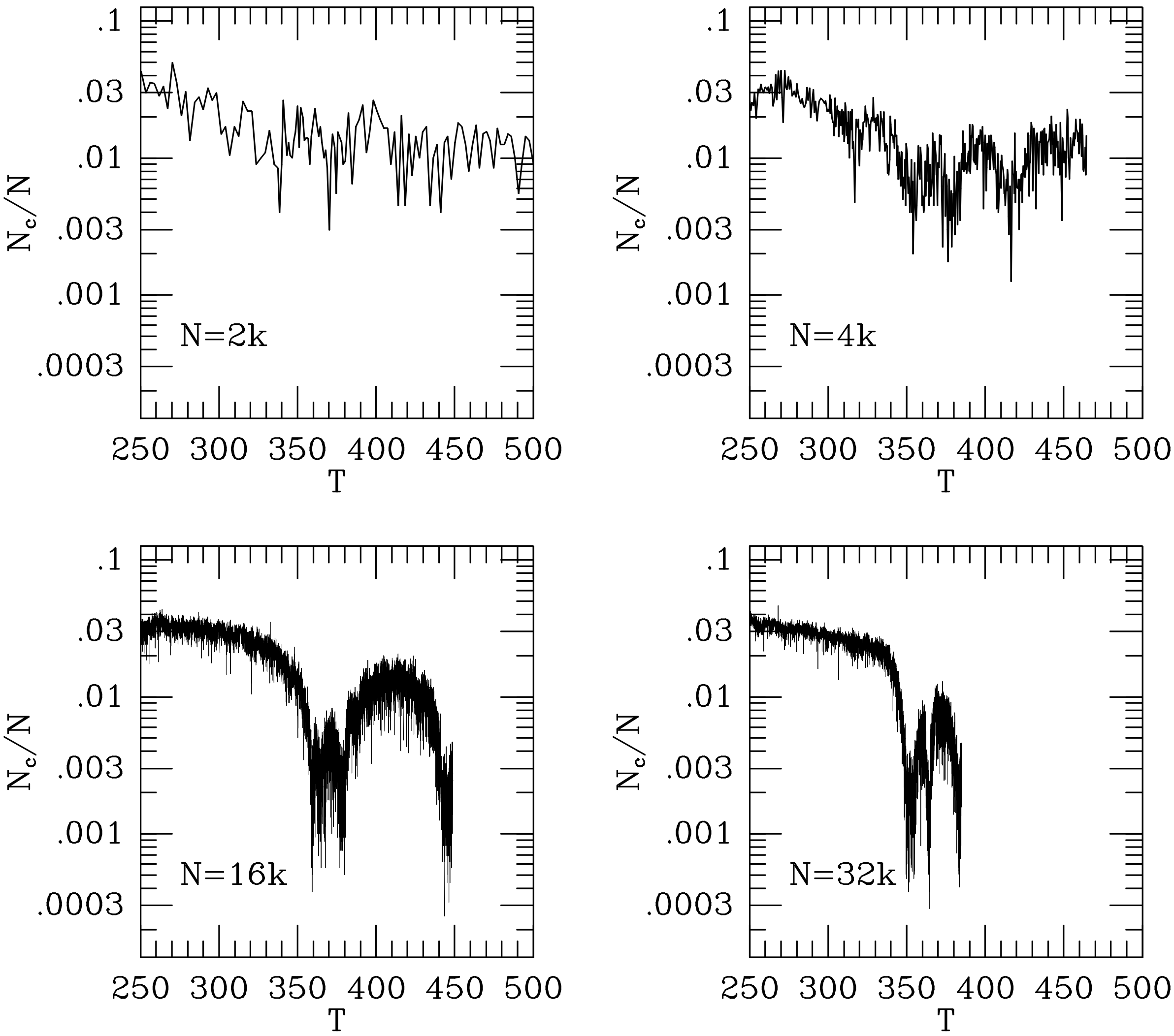


N=32k
N=16k
N=8k
N=4k
N=2k
0
5
10
15
0
200
400
600
$\log\rho_c$
Scaled N–body Time

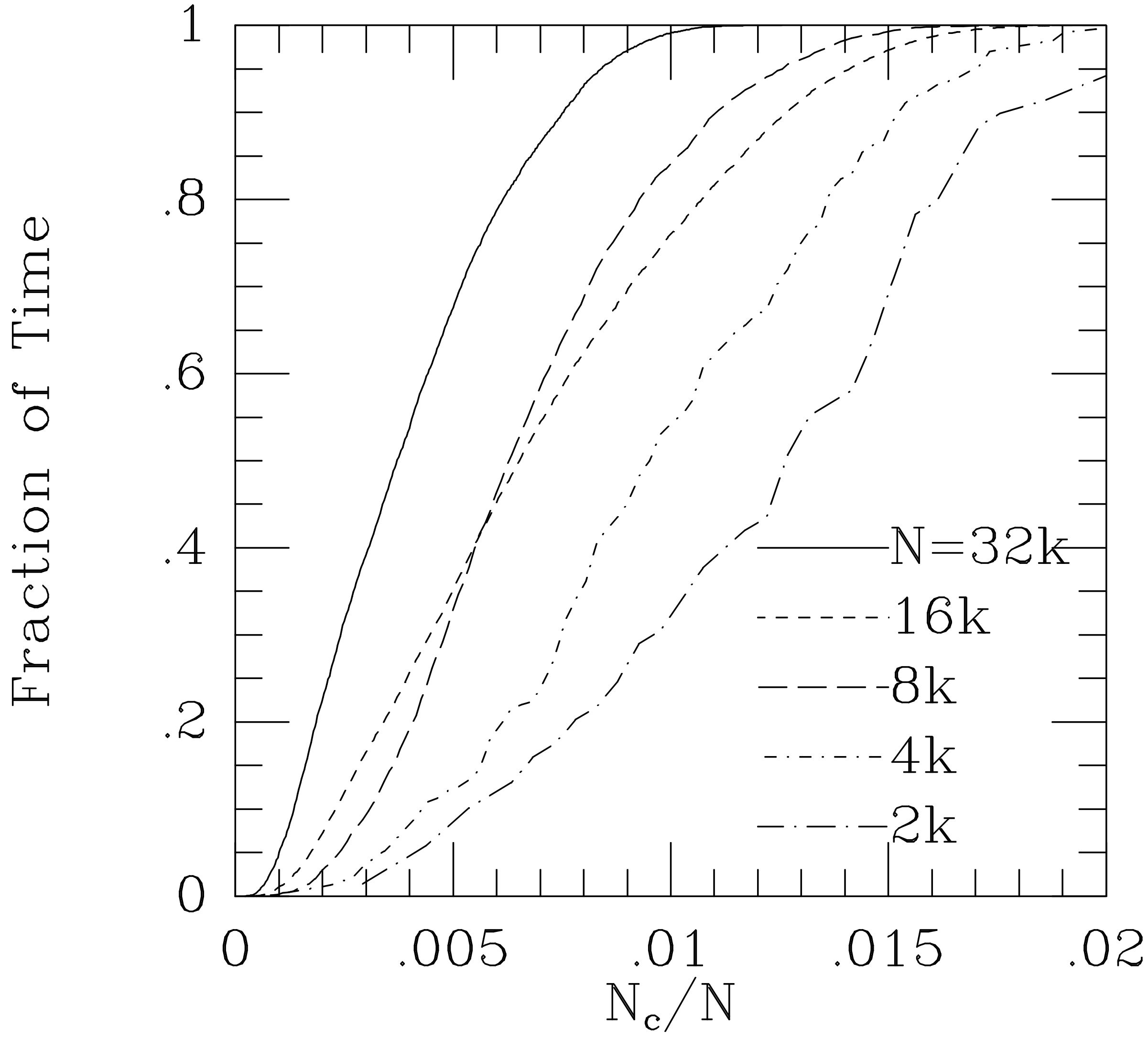


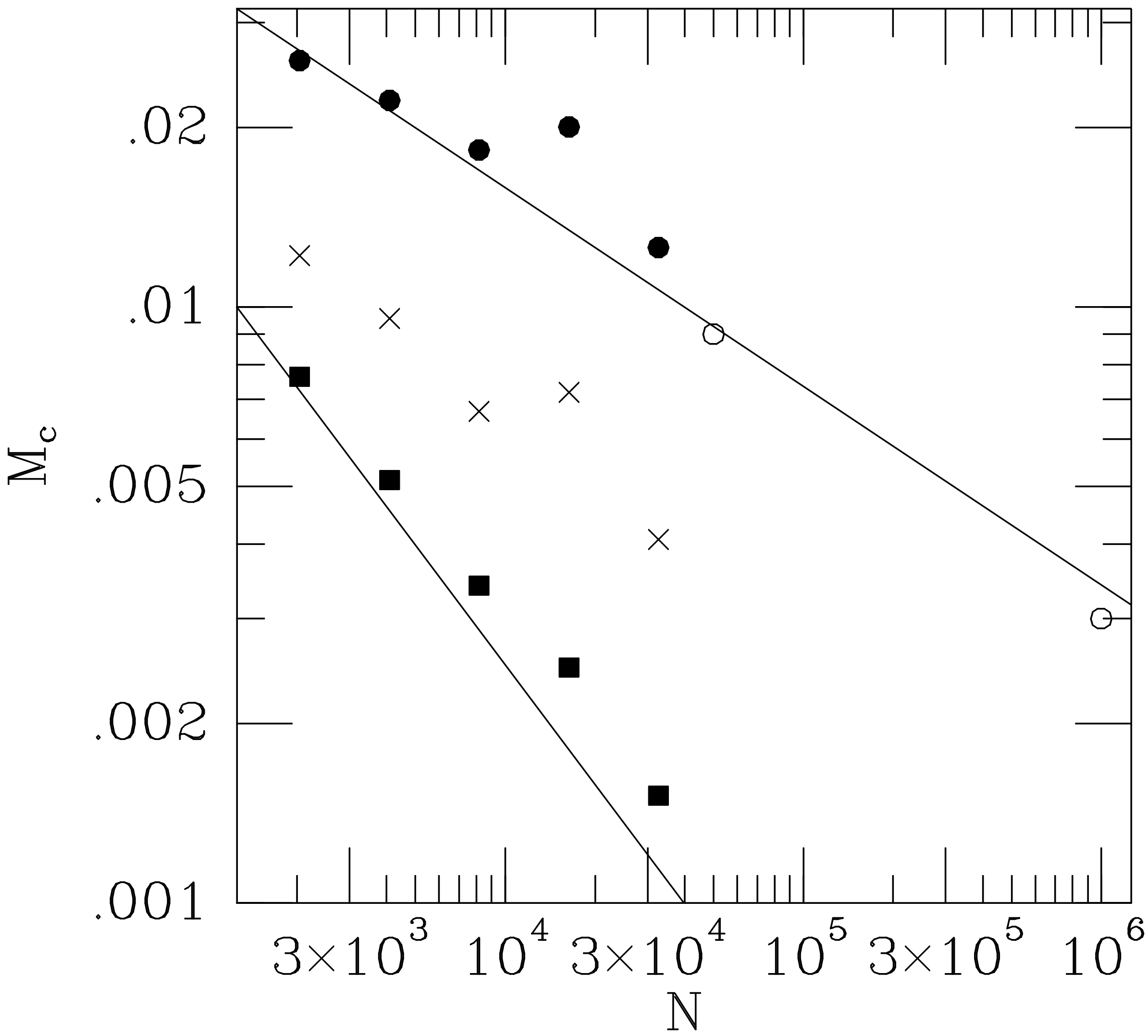
log r_c
−1.5
−2
−2.5
−3
−3.5
6500
7000
7500
E_b/E_{total}
.6
.4
.2
0
6500
7000
7500
T
log $<v^2>$
1.6
1.4
1.2
1
−3
−2
−1
log r
log $<v^2>$
1.4
1.3
1.2
1.1
1
−3
−2
−1
log r


1.6
1.4
1.2
1
.8
1
2
3
4
5
log v_c^2
log ρ_c
A
B



N=2k
N=4k
N=16k
N=32k
N_c/N
T
.1
.03
.01
.003
.001
.0003
250
300
350
400
450
500


Fraction of Time
1
.8
.6
.4
.2
0
0
.005
.01
.015
.02
N_c/N
N=32k
16k
8k
4k
2k


.02
.01
.005
.002
.001
M_c
3×10^3
10^4
3×10^4
10^5
3×10^5
10^6
N