N-body (and SPH) simulation on GRAPE-6

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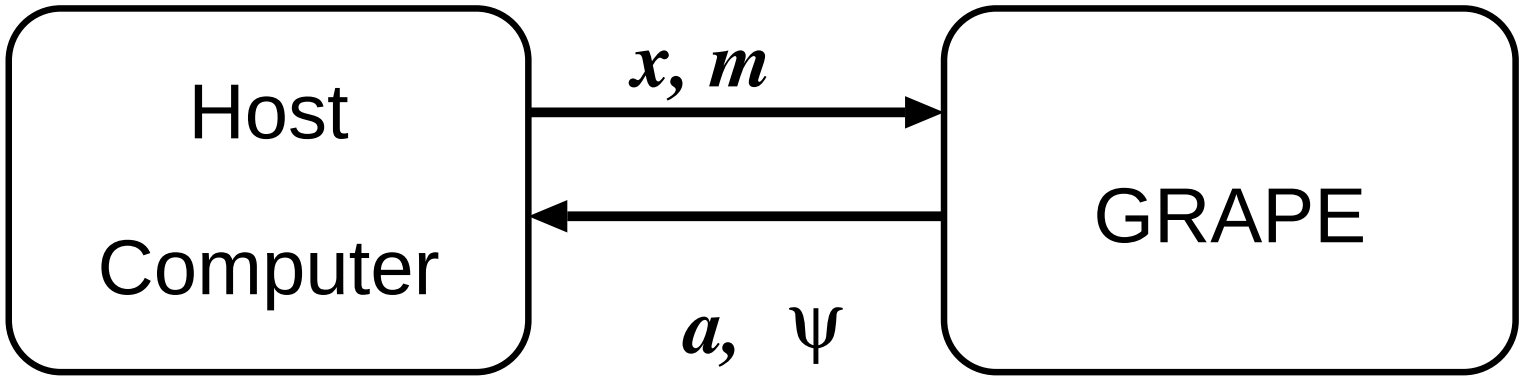
- **Basic Idea of GRAPE**
- **History of GRAPE Project**
- **GRAPE-4**
- **GRAPE-6**
- **Simulation of Binary BH in galactic cores on GRAPE-4/6**

Basic idea of GRAPE

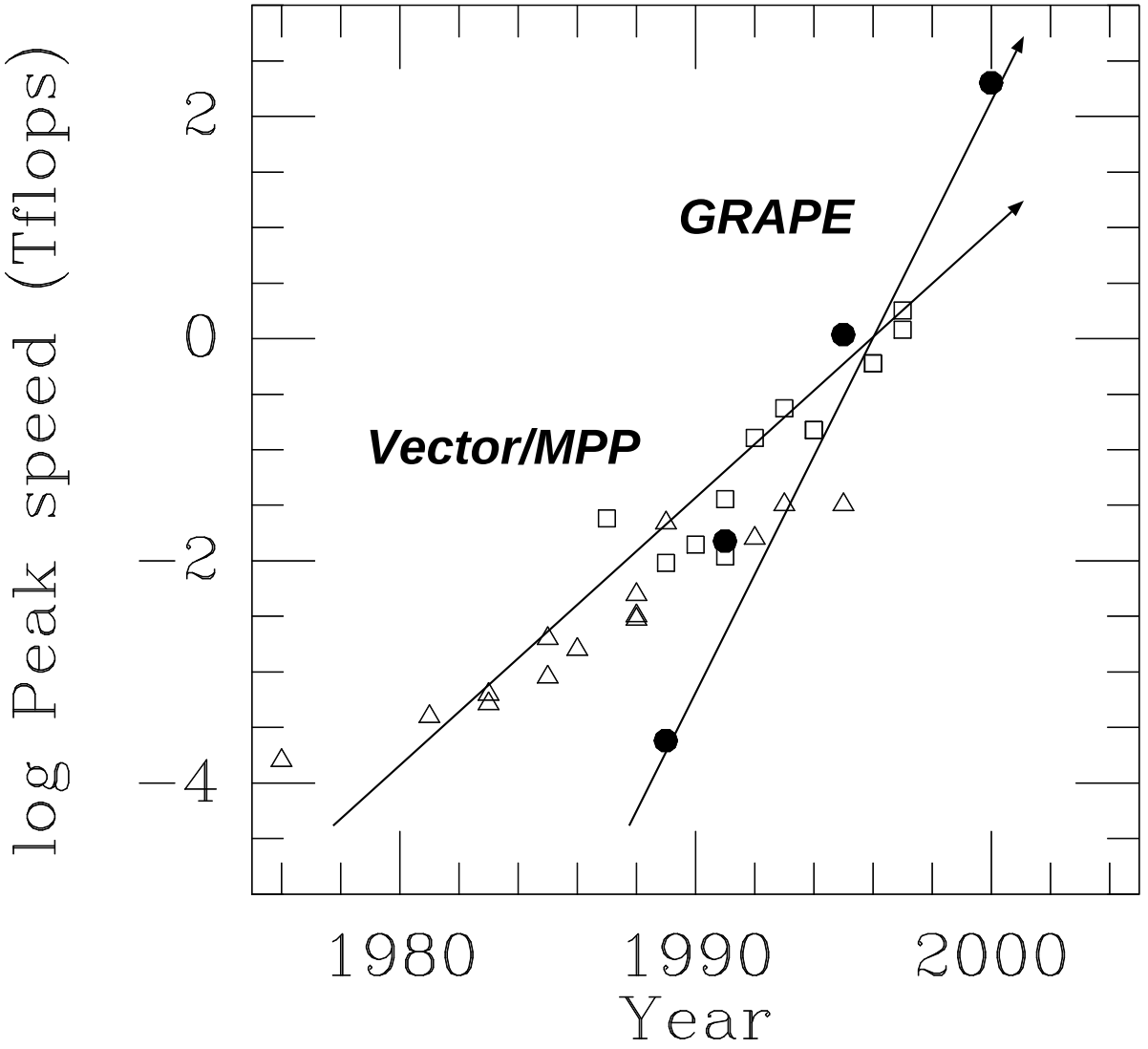
Special-purpose hardware for force calculation
General-purpose host for all other calculation



Advantages of GRAPE

- Special-purpose hardware
  - Hardwired logic → No software needed
  - Pipelined processor → High performance
- General-purpose host
  - Programmable in high-level languages
  - existing programs can be used with "small" modifications
  - can be used with Barnes-Hut tree/Aarseth scheme

Evolution of GRAPE systems



History of the project

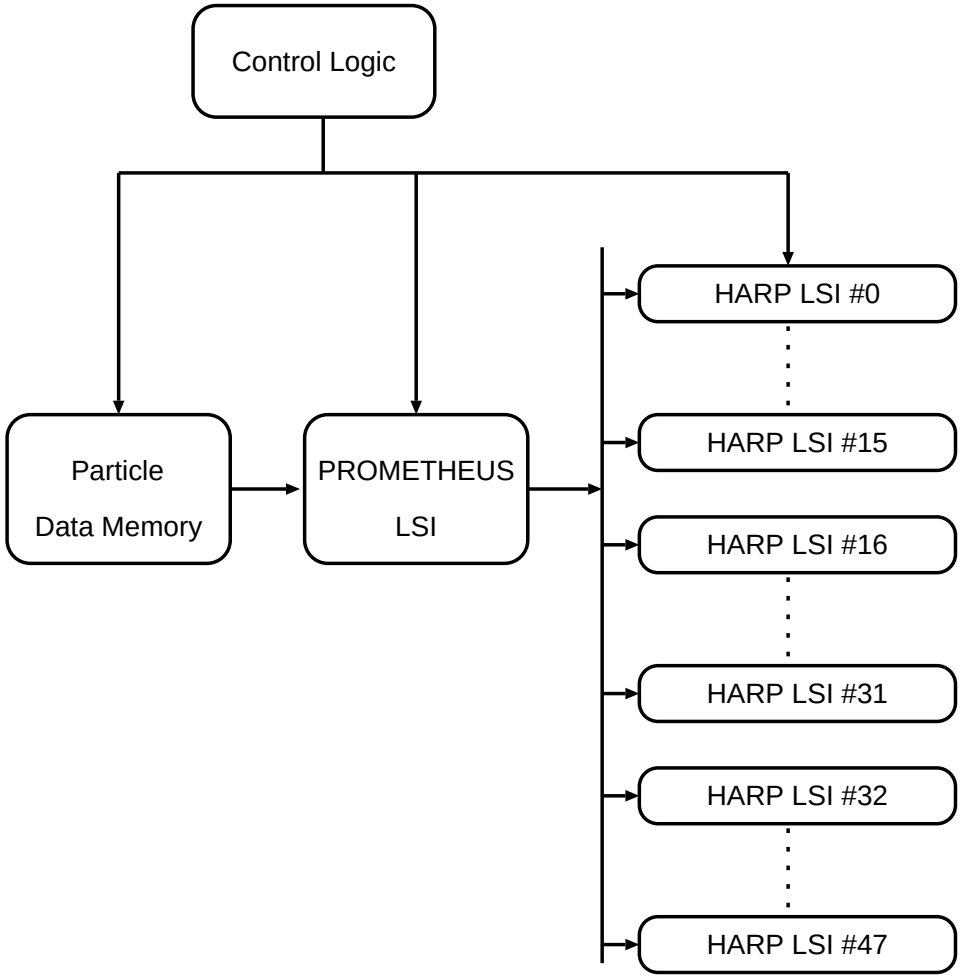
Name	development period	speed	Accuracy	Note
GRAPE-1	89/4 — 89/10	120 M	L	
GRAPE-2	89/8 — 90/5	40 M	H	
GRAPE-1A	90/4 — 90/10	240 M	L	
GRAPE-3	90/9 — 91/9	14 G	L	custom chip
GRAPE-2A	91/7 — 92/5	180 M	H	
HARP-1	92/7 — 93/3	180 M	H	Hermite scheme
GRAPE-3A	92/1 — 93/7	6 G	L	∼100 boards made
GRAPE-4	92/7 — 95/7	1T	H	8 sites
MD-GRAPE	94/7 — 95/4	4G	H	programmable interaction
GRAPE-5	96/4 — 99/4	40G	L	
GRAPE-6	97/8 —	100T	H	under development

GRAPE-4

Features:

- Predictor pipeline in hardware (for individual timesteps)
- Massively parallel system with 1692 pipeline chips
- 36 processor boards each with 47 processor chips
- calculates force on 94 particles
- Peak speed: 1.08 Tflops

Structure of GRAPE-4 processor board



48 pipeline LSIs share one memory unit

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Massively parallel system with small parts count (=low cost)

GRAPE-6

Basic idea

Technology update for GRAPE-4.

- 4 times more transistors in every 3 years
- 2 times faster clock speed in every 3 years

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New machine to be completed 5 years after GRAPE-4 can be 32 times faster.

Project goals for GRAPE-6

- Peak speed $>$ 100Tflops (We've got somewhat more money...)
- Improved communication speed ($>$ 2GB/s)
- Minor fixes for some "features" of GRAPE-4...

Overall structure

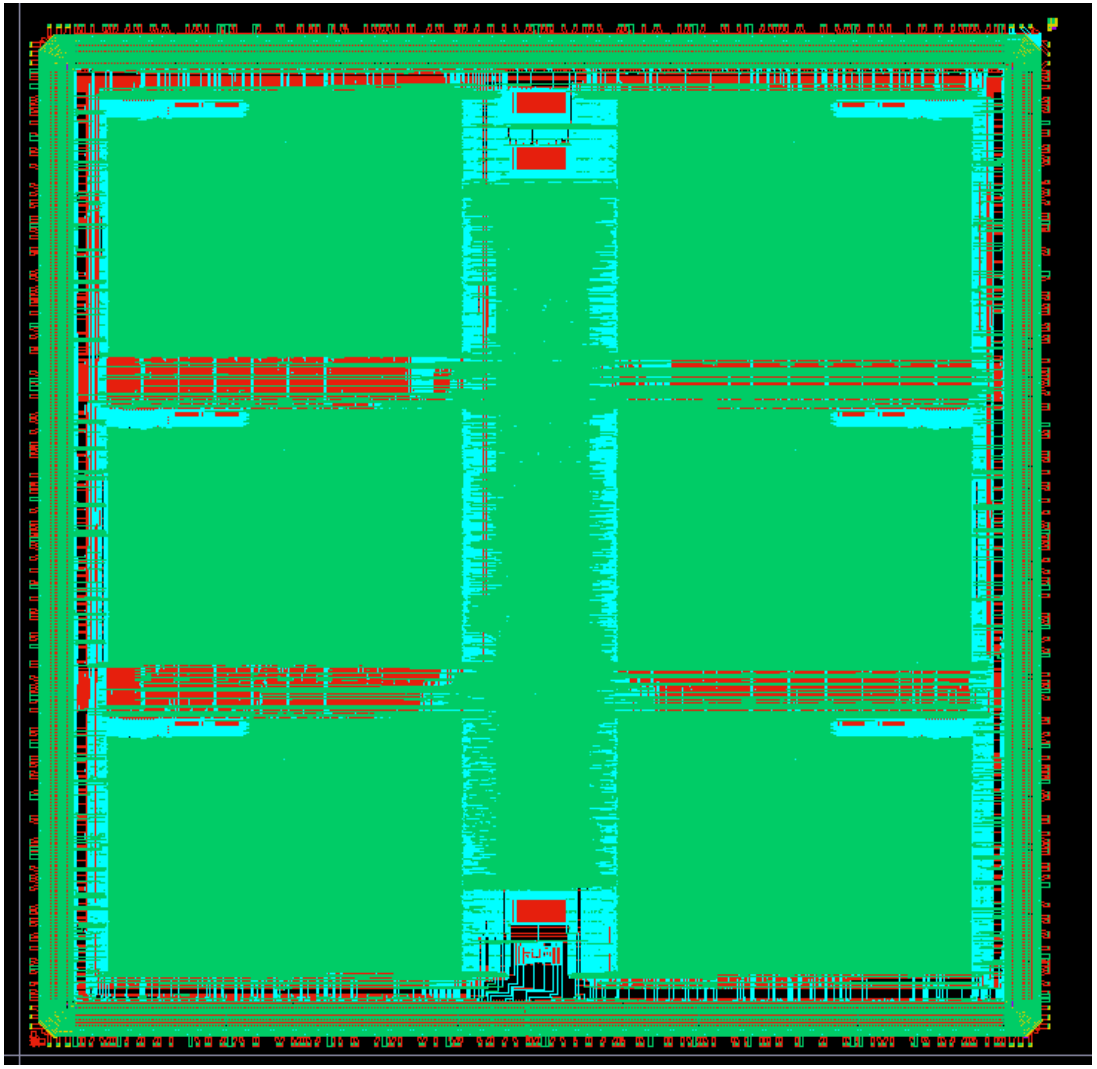
- 16 (or somewhat less...) clusters
- 16/8 processor boards per cluster
- 8 processor modules per board
- 2/4 chips per modules
- 6 pipelines per chip
- 60 arithmetic units per pipeline

In total,1.5×10^6 Arithmetic units operating at 90 MHz = 135 Tflops

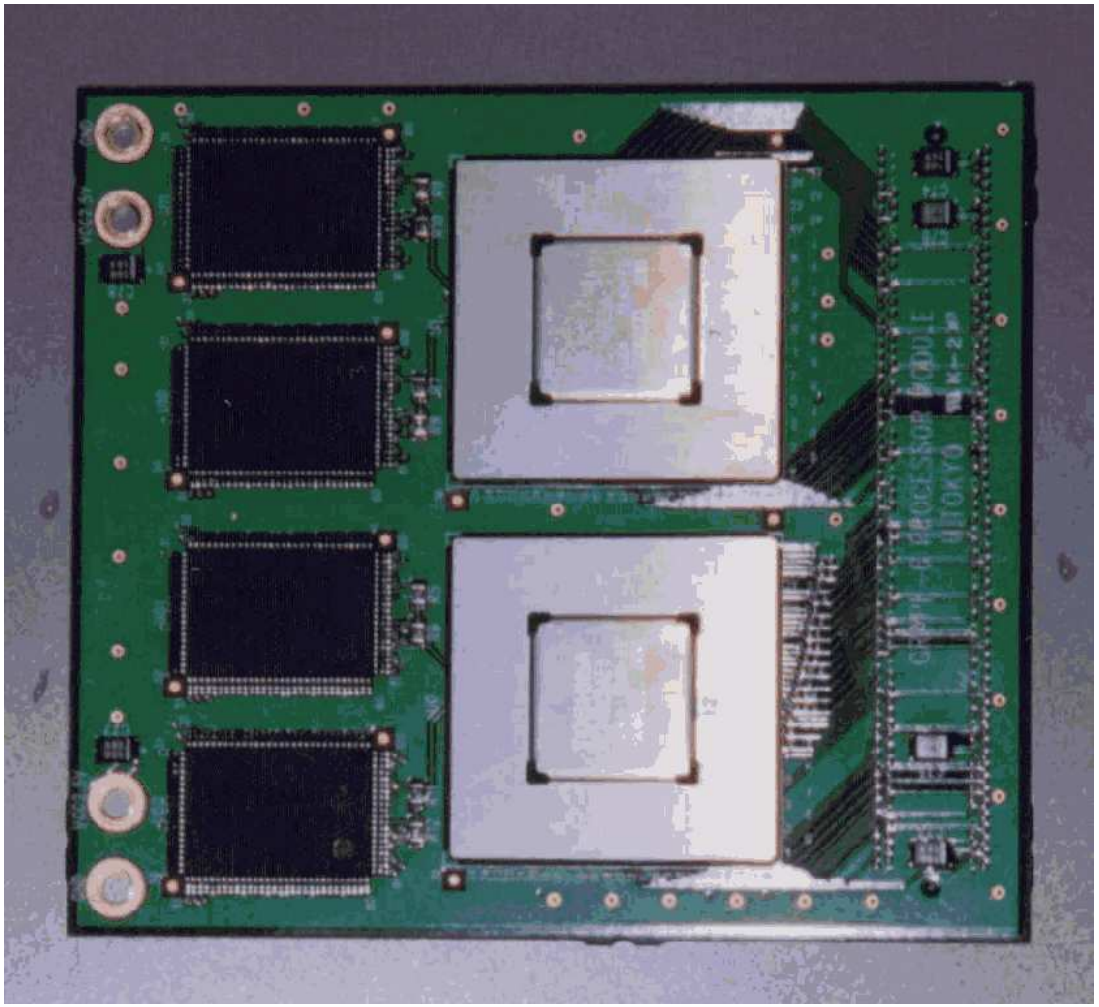
GRAPE-6 Processor Chip

- 0.25 μm custom design (Toshiba TC240E, 1.8M gates)
- 2.5 V power supply, 100 MHz operation (design goal)
- 6 pipelines in a chip
- $>$ 30 Gflops (equivalent) per chip
- system clock: 25 MHz
- Single chip equivalent to single GRAPE-4 board

GRAPE-6 Processor Chip

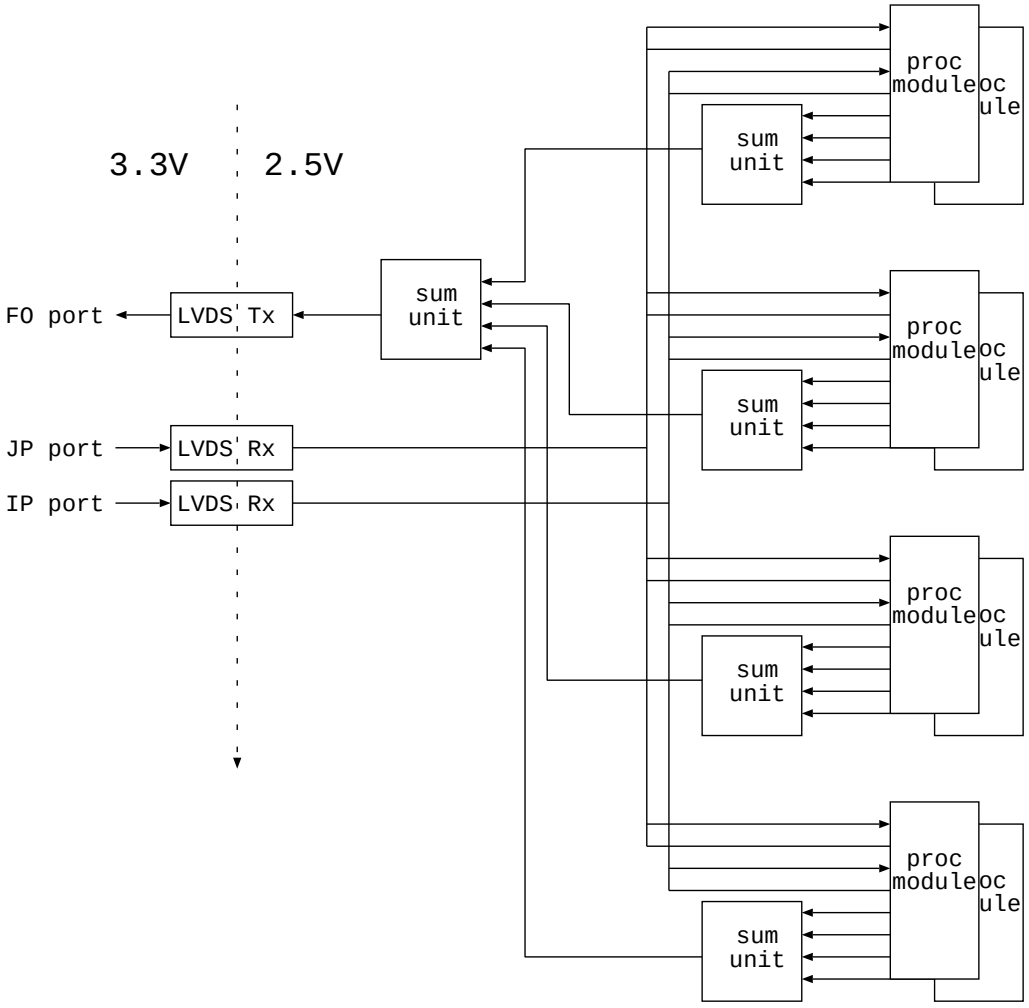


GRAPE-6 processor module



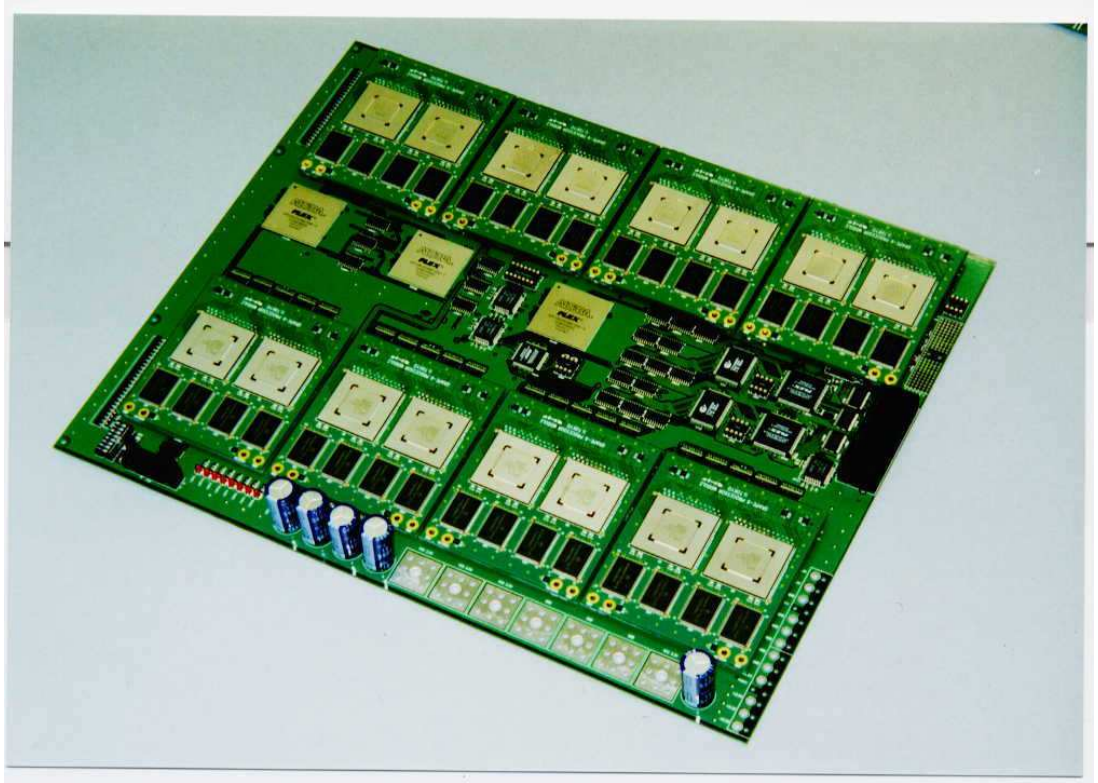
- Two processor chips + Memory chips.
- Integrates all functions of GRAPE-4 processor board.

The GRAPE-6 Processor board



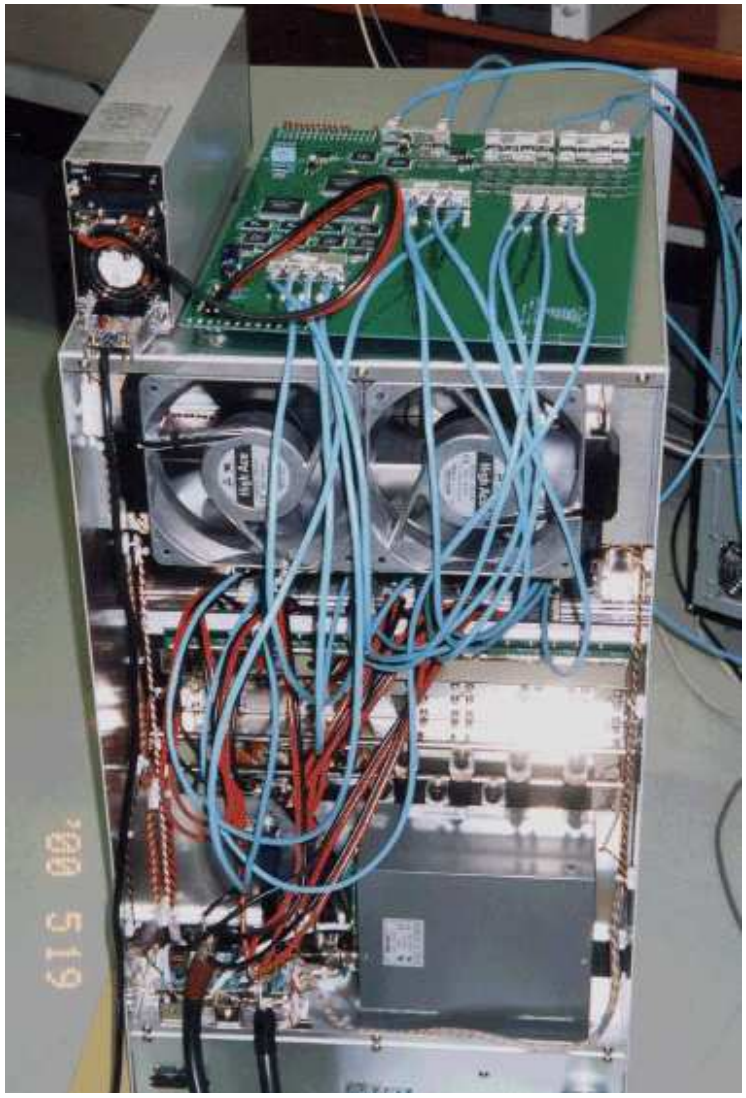
- Single board with 16-chips
- Semi-serial (LVDS) interface (350MHz clock, 4 wires)
- Two input ports and one output port
- Result from 16 chips are summed by a reduction tree

The GRAPE-6 processor board



GRAPE-6 cabinet and 4-board system

Current status



- 4-board, 2-Teraflops system is now up and running.
- Current clock: 88 MHz
- Number of chips: 64
- Full 100-Tflops system should be ready by the end of 2001

N-body simulation on GRAPE-6

Thermal (long-term) evolution

GRAPE-4: can do equal-mass 32k-particle globular cluster simulations in weeks
GRAPE-6: can handle 150k (equal-mass) or 300-500k for realistic system.

Dynamical evolution of dense stellar system

GRAPE-4: up to about 300K
GRAPE-6: up to 10M in direct method

Galactic dynamics or cosmology with treecode

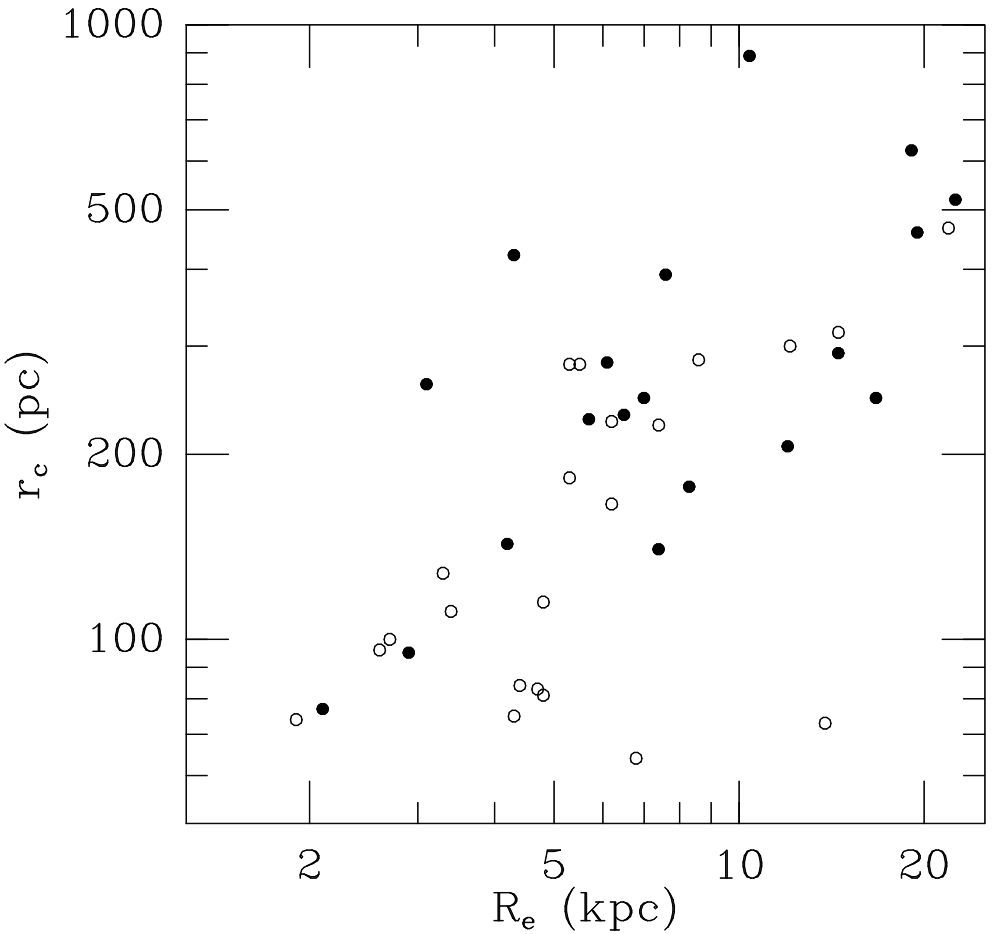
GRAPE-4: 10M (host memory limited...)
GRAPE-6: 1G

Massive black hole binary in galactic core

The central structure of ellipticals has been a mystery

The pre-HST view (Data from Lauer 1985)

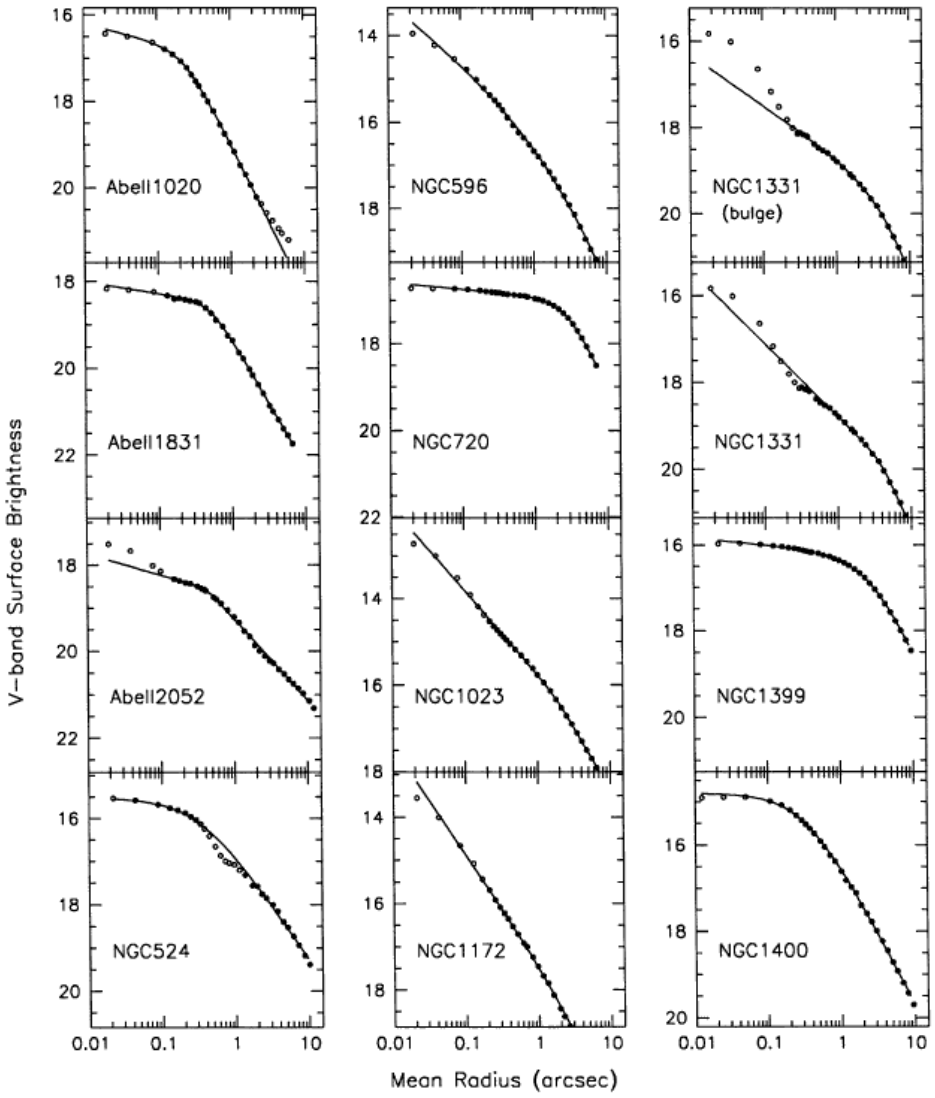
- Ellipticals have cores
- $r_c \propto R_e$



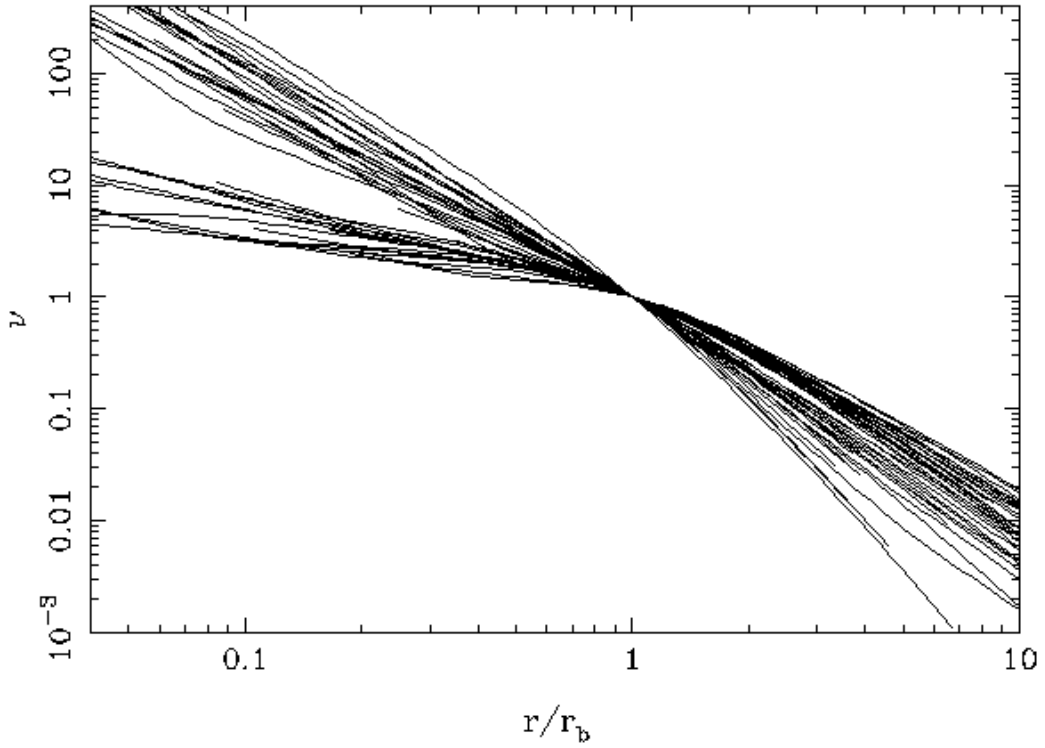
The HST view

Byun et al. (1996)

No observed elliptical showed a real core...



Deprojected profiles.



Gebhardt et al (1996)

- Cuspy galaxies
  — $\rho \propto r^{-2}$
- "Weak cusp" galaxies
  — $\rho \propto r^{-0.5 \sim -1}$

What have been recognized as "cores" are are all week cusps.

How are these "weak cusp" ellipticals formed?

Theories predict...

- No core — dissipational collapse
- r_c = const. — merging

No theory predicts large cores.

Massive central BHs do not help

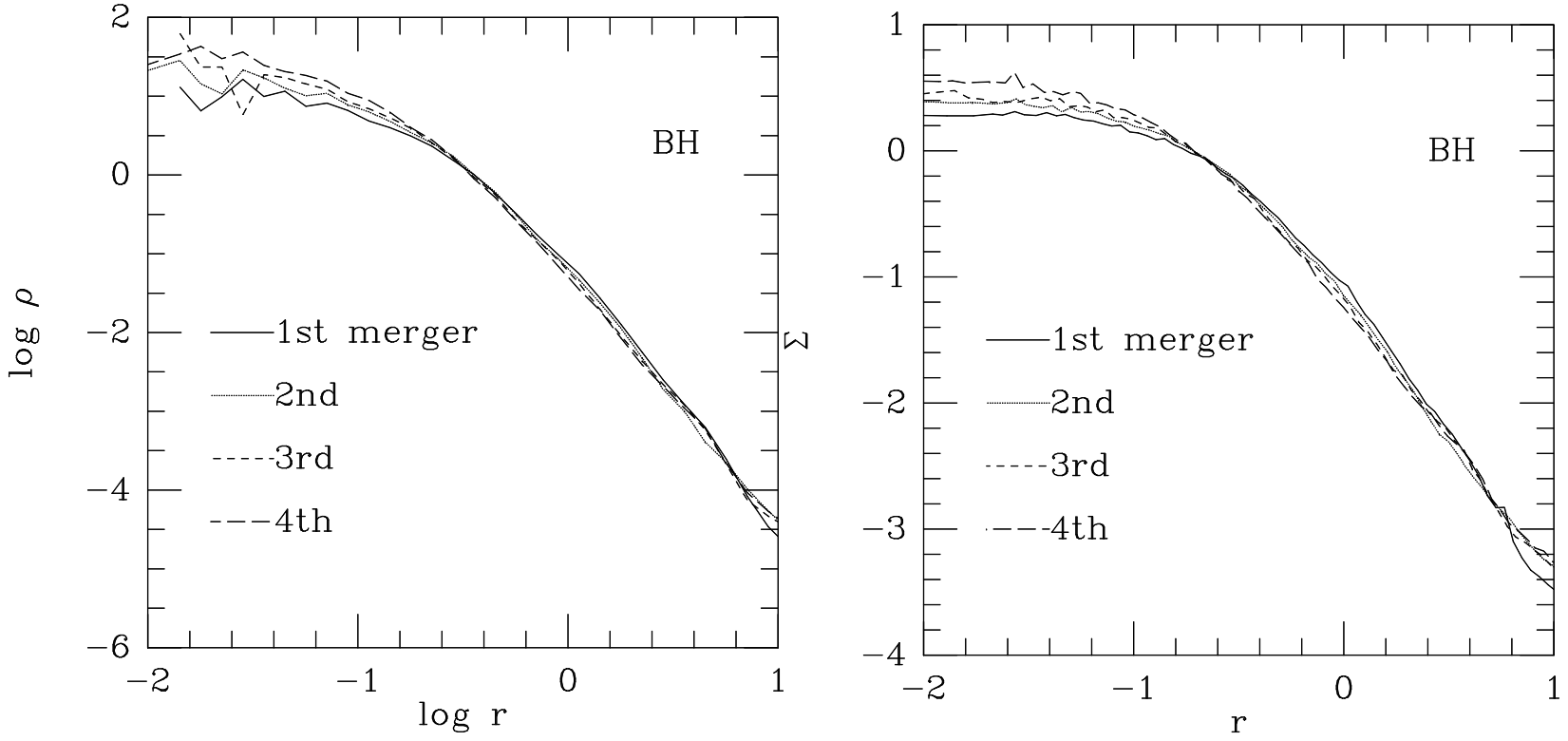
- Thermal evolution (Bahcall & Wolf 1976) forms $\rho \propto r^{-7/4}$
- Adiabatic growth of the central BH (Young 1980) forms $\rho \propto r^{-3/2}$

Assumptions behind theoretical predictions: BH grows at the core.

Merging with central BH

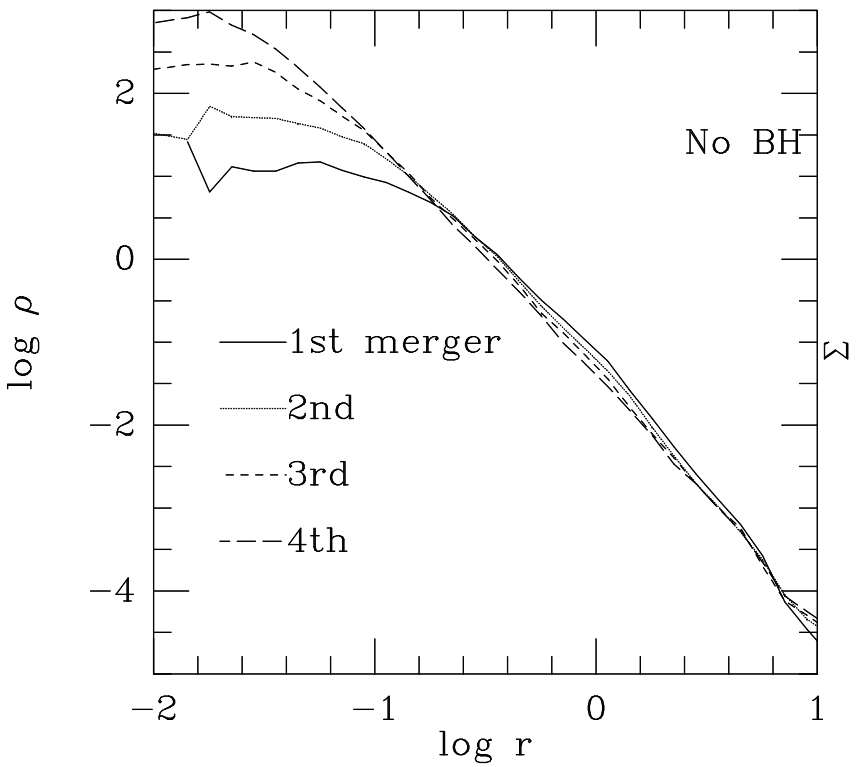
What if BHs come from outside?

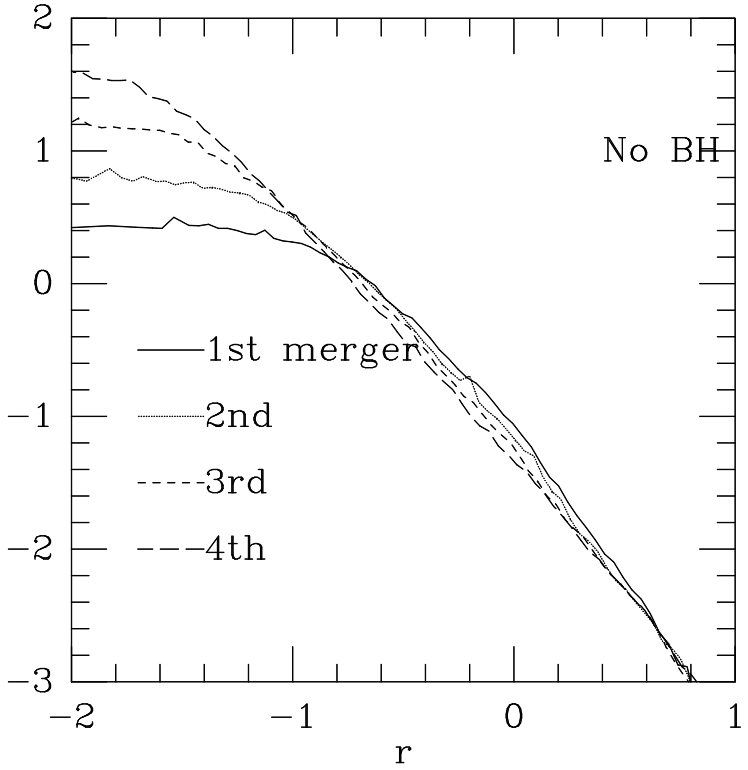
= What if two galaxies with BHs merge?



Makino & Ebisuzaki (1996, GRAPE-4)

Without central BH...





Simulation result summary

- Merging of galaxies with central BHs results in "weak cusps"
- $r_c/R_e \sim M_{BH}/M_g$

Seems to explain observation rather well.

Further study on the formation mechanim of the weak cusp

Nakano and Makino 1999a (Numerical) and 1999b (Analytical)

BH spiral-in to the center in dynamical timescale inevitably results in the weak cusp.

Fate of BH binary

BH binary

—Shrink and finally merge?

—Throw out all nearby stars and become stable?

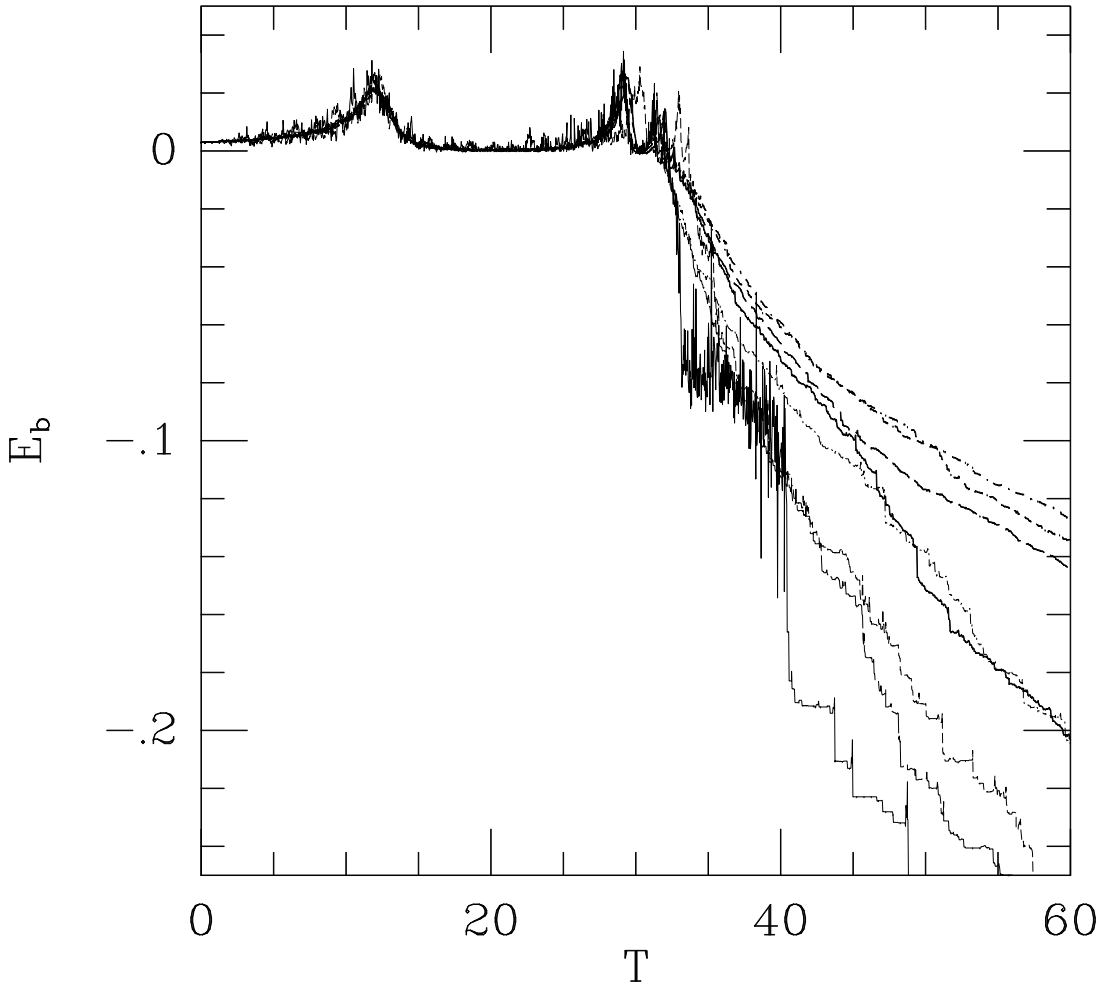
- Kicked out of galaxy by triple interaction? (further merging with other galaxy)

(Begelman, Blandford and Rees 1980)

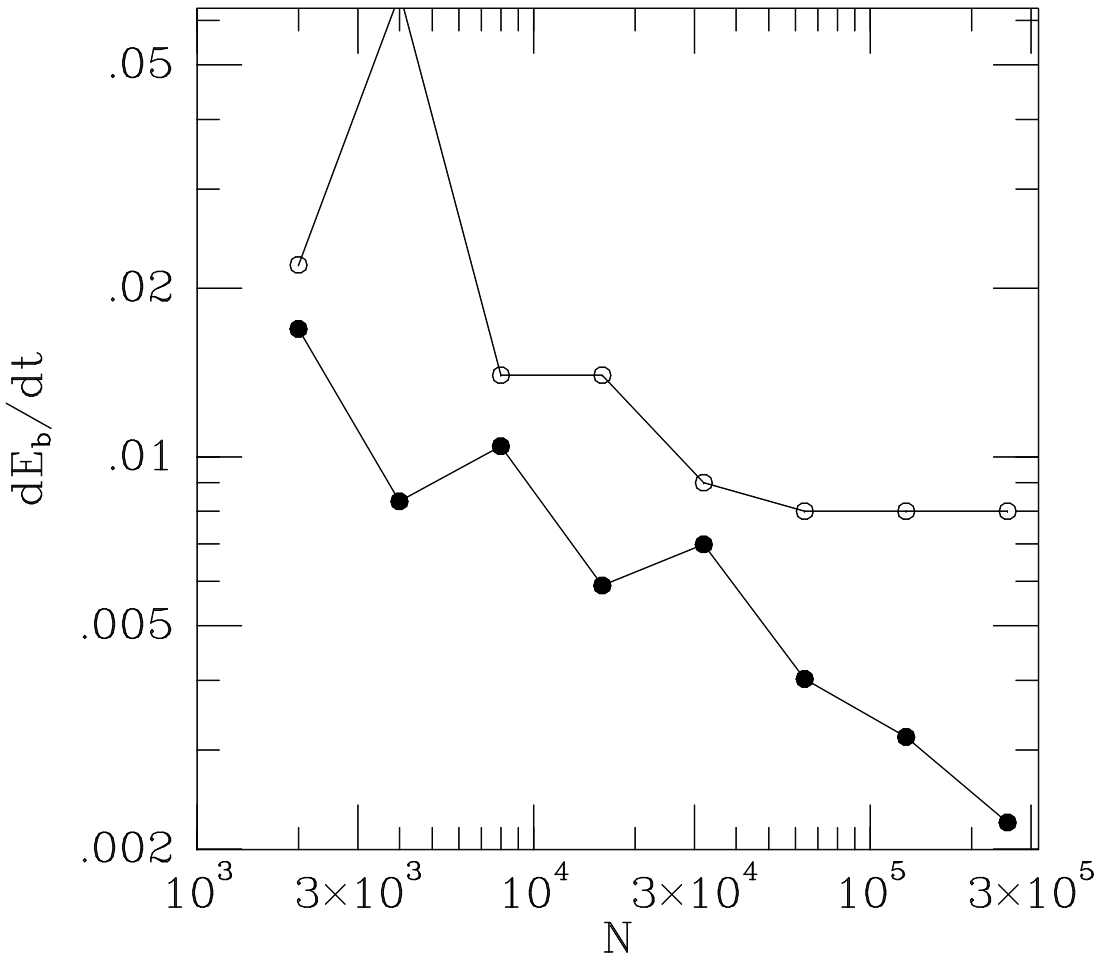
Numerical studies

N is too small — try to see the N-dependence.

Result for N=2k — 256k (Makino 1997, GRAPE-4)



Growth rate



Open: early
Filled: late

Timescale:

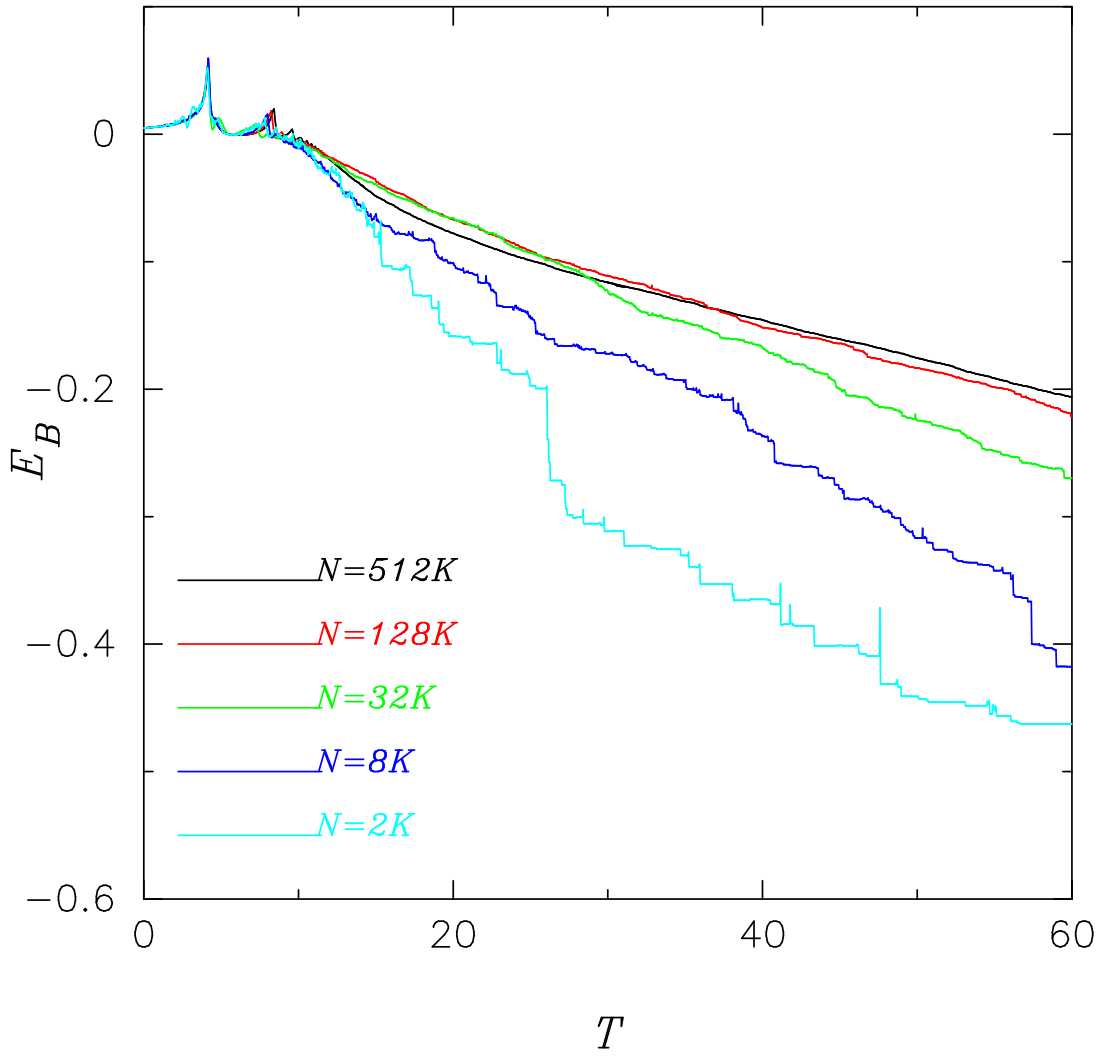
Early phase: converges?
Late phase: $\propto N^{1/3}$?

GRAPE-6 result (preliminary...)

Currently, similar calculations with N up to 1M are underway.

Result for N up to 512k

Consistent with GRAPE-4 result (so far..)



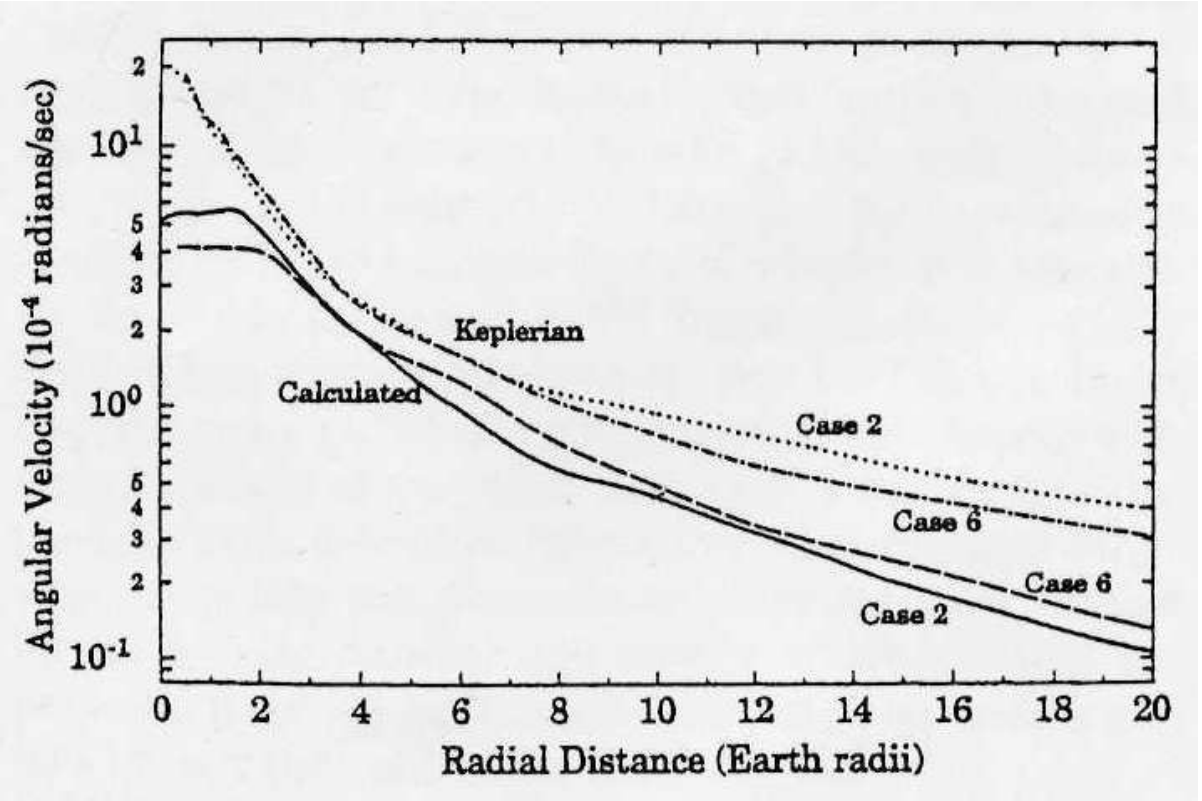
Result

- Evolution timescale depends on N (number of particles used)
  - However, the dependence is much weaker ($t \sim N^{1/3}$) than theoretical prediction ($t \sim N$).
- Extrapolation to large N: BHs do not merge within Hubble time
- Larger simulations on GRAPE-6 is currently underway.

SPH Simulation of "Giant Impact"

Preliminary simulation, not fully analyzed.

Cameron's SPH calculation (1997): Giant impact would not result in a moon because: rotation too slow



Initial model parameters 1: EOS

Equation of State:

$$\frac{P}{\rho} = (\gamma - 1)u_k + \frac{n}{3}\rho^{n/3}$$

First term: ideal gas
Second term: polytropic (we used $n = 12$)

This EOS implies we neglect the attracting term of the intermolecular potential, which is a good approximation if the temperature is high.

We chose this EOS for SIMPLICITY, to compare the results of SPH calculations with different set of parameters.

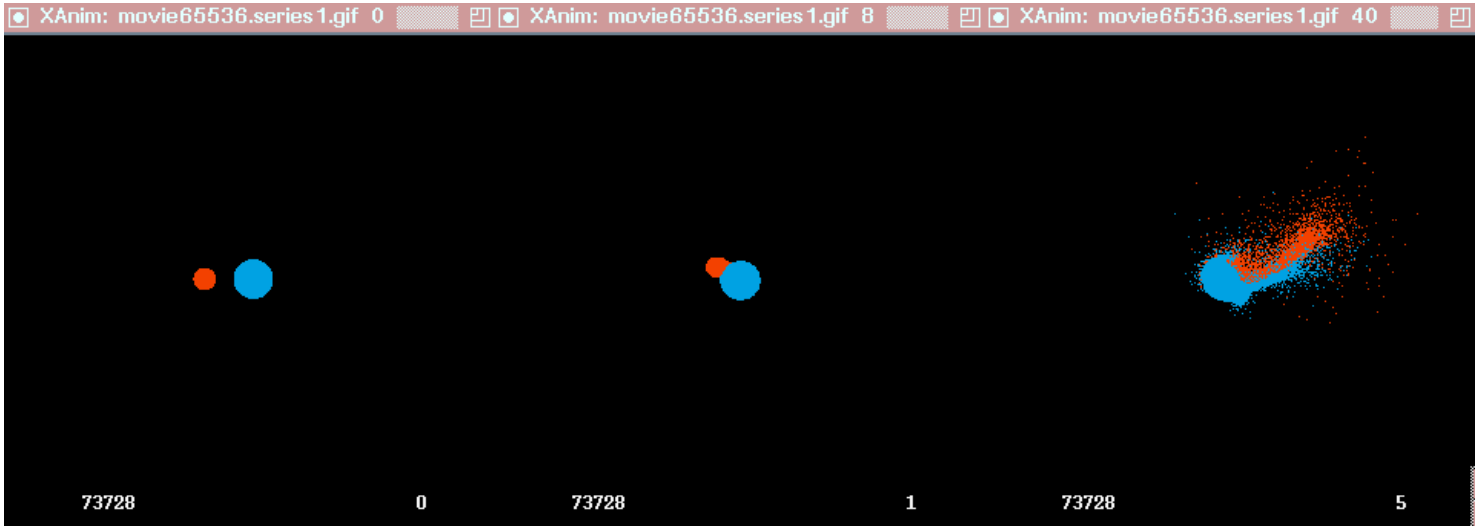
Initial model parameters 2

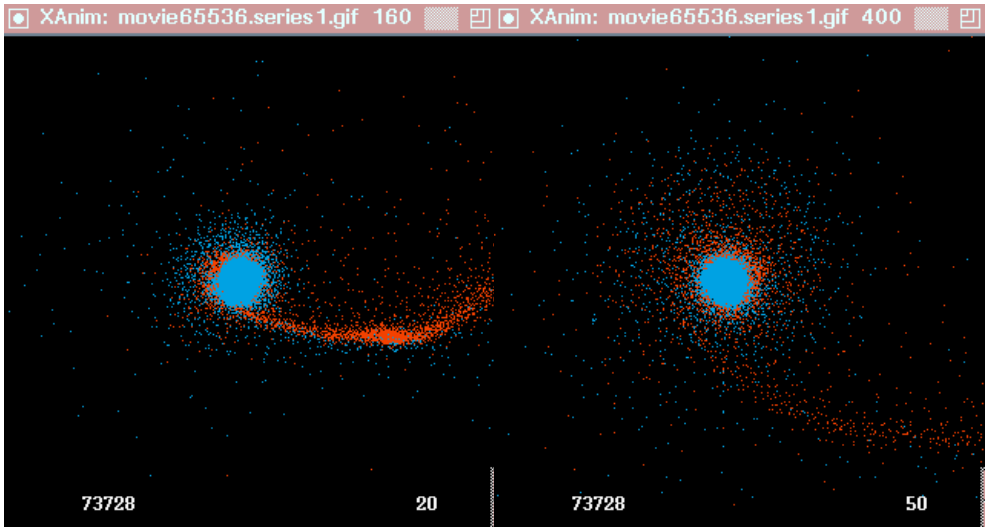
Initial orbit: parabolic
Mass ratio: 1:8
Orbital angular momentum: 2, 1, and 1/2 J_0
(J_0: present Earth-moon system)

Number of particles: 1152 — 73728

System of Units:
$M = 1$, $R = 0.81$, $G = 1$, 1 time unit $\sim$ 1100 sec

Result samples





Current status

- Simulations with large N seem to support Cameron's (1997) result.
- However, the effect of numerical viscousity dominates the final rotation profile.

Summary

- GRAPE-6 is currently under development.
- 64-chip system (2 Tflops peak) is up and running.
- >100-Tflops full system will be ready by 2001.
- Star-by-star simulation of (small) globular clusters is comming soon.

SPH

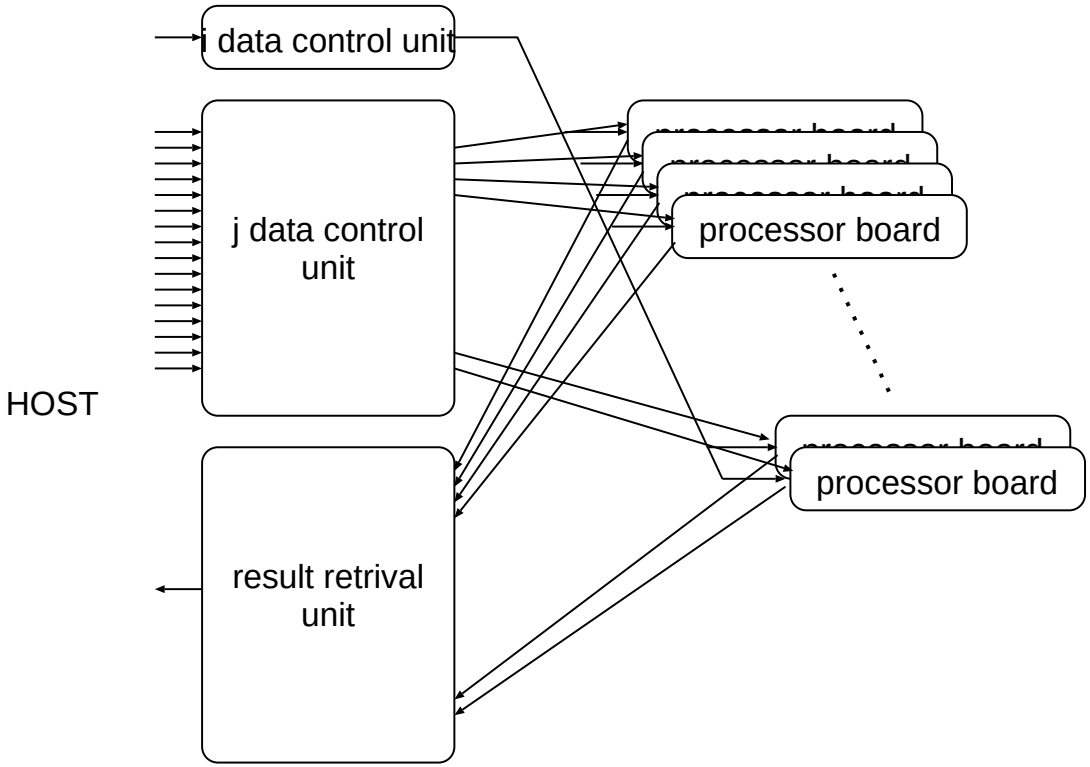
Particles interact through hydrodynamical interaction as well as gravity.

Timescale is (in most applications) dynamical.

SPH interaction is short-range interaction.

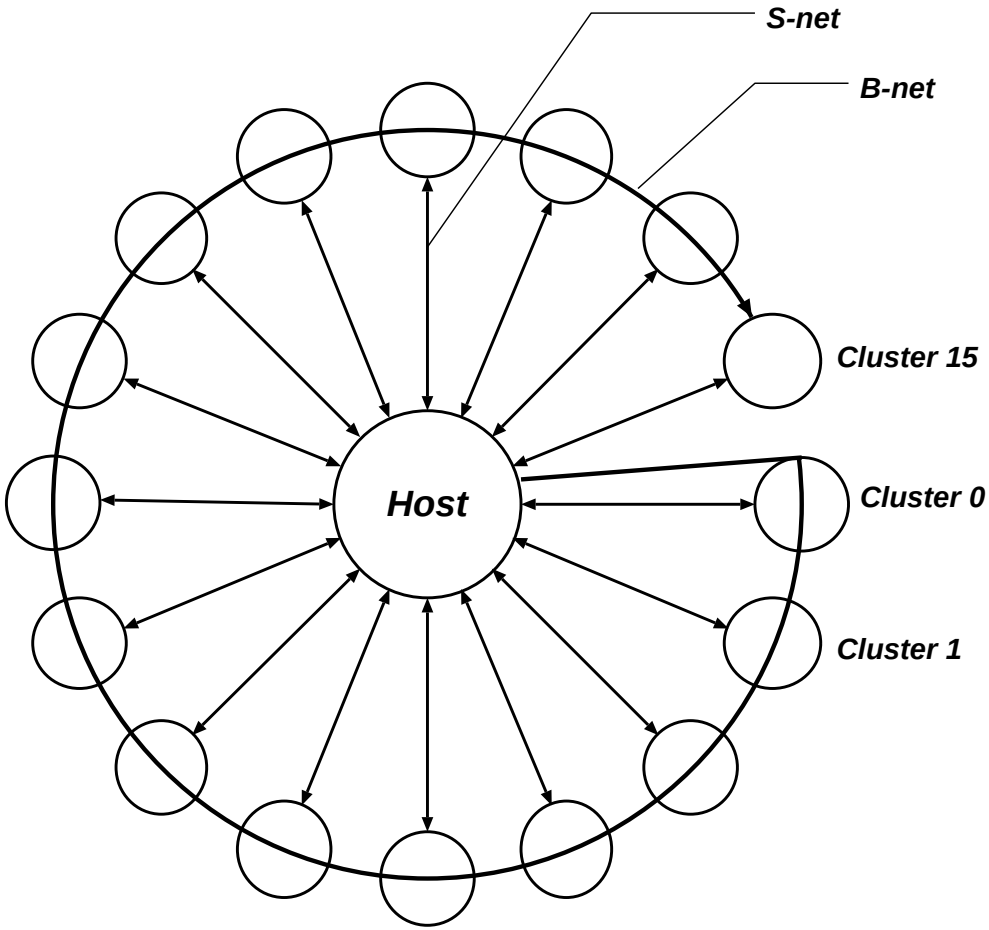
Calculation cost of SPH is less than that of gravity. (but not by a very large factor...)

Structure of a cluster



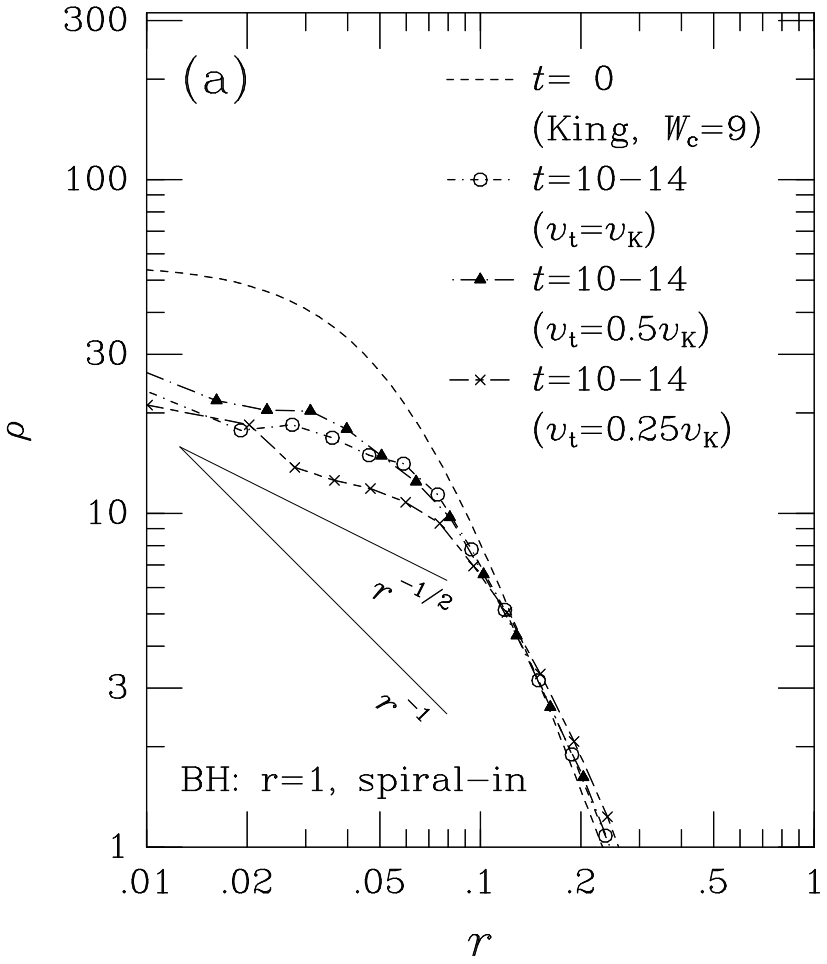
- 16 (or 8) processor boards
- 3 dedicated communication networks

Structure of the total system



- 16 clusters
- 256 chips/cluster
- S-net for register read/write
- B-net for particle memory write (multicast)

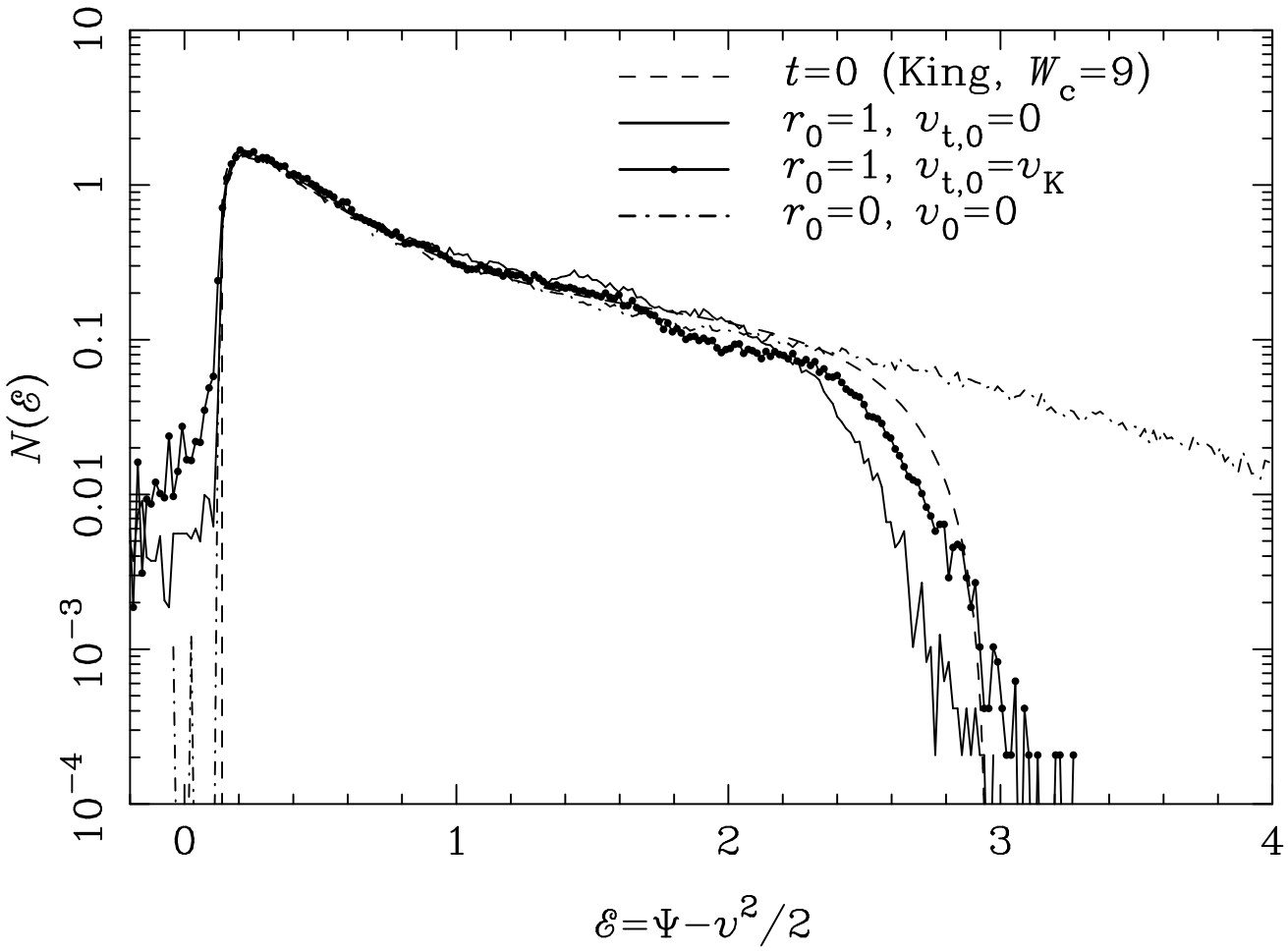
Simple Experiment



Let one single BH fall down to the center of a spherical galaxy (Nakano and Makino 1999a)

The same weak cusp was formed, but WHY?

Energy distribution function



Cusp and $N(\mathcal{E})$

Energy distribution function is practically unchanged

$\downarrow$

$N(\mathcal{E})$ becomes zero at finite $\mathcal{E} = \mathcal{E}_0$.

No star is strongly bound to the central BH.

$\downarrow$

$f(\mathcal{E})(= N(\mathcal{E})/A(\mathcal{E}))$ also becomes zero at finite E.

This explains why there is a weak cusp.

Theory for the weak cusp

$f(\mathcal{E}) \rightarrow \rho(r)$ (Nakano and Makino 1999b)

$$\rho(r) = 4\pi \int_0^{\Psi(r)} f(\mathcal{E}) \sqrt{2\left[\Psi(r) - \mathcal{E}\right]} d\mathcal{E},$$

If $\Psi(r) > \mathcal{E}_0$, the above can be rewritten as

$$\rho(r) = 4\pi \int_0^{\mathcal{E}_0} f(\mathcal{E}) \sqrt{2\left[\Psi(r) - \mathcal{E}\right]} d\mathcal{E}.$$

In the region $\Psi(r) > \mathcal{E}_0$,

$$\rho(r) = 4\sqrt{2}\pi \sqrt{\Psi(r)} \int_0^{\mathcal{E}_0} f(\mathcal{E}) \left[1 - \frac{1}{2}\frac{\mathcal{E}}{\Psi(r)} + O\left(\left[\frac{\mathcal{E}}{\Psi(r)}\right]^2\right)\right] d\mathcal{E}$$

$$\rho(r) \propto \sqrt{\Psi(r)} \sim \sqrt{\frac{GM_{\rm BH}}{r}}.$$

N-body simulations

N-body simulations

Most straightforward way to study the dynamics and evolution of stellar systems.

Two types of systems/simulations

- "Collisionless" systems
  - two-body relaxation effect is small
- "Collisional" systems
  - two-body relaxation drives the evolution of the system

Required accuracy and time-integration scheme are very different.

Limit for the accuracy/reliability

Main limitation to the accuracy = Number of particles

- resolution
- numerical relaxation (collisionless)
- time scaling problem (collisional)

Number of particles is limited by:
speed of computation.

How can we speed up the calculation

Two options:

- Develop better algorithms.
- Use fast computers (if necessary, adapt algorithms for new hardware).

Yet another approach:

- Develop fast computers = GRAPE

New architecture

- each chip has its own memory
- use fast clock for memory
- pipelines on a chip share the memory
- different chips in a cluster calculate the force on the same particles

↓

- reasonable performance with blockstep scheme (48 virtual pipelines per chip)

Summary for BH binary

- We found the weak cusp is formed in merging of galaxies with massive BHs.
- The formation mechanism of the weak cusp is explained by simple analytical argument.
- Weak cusps are signatures of the merging of galaxies with BHs
- The merging timescale of BH binary may be very long. Typical large ellipticals may host binary BHs.